

LECTURE 5. NON-CUPRATE SUPERCONDUCTIVITY

1. Alkali Fullerenes

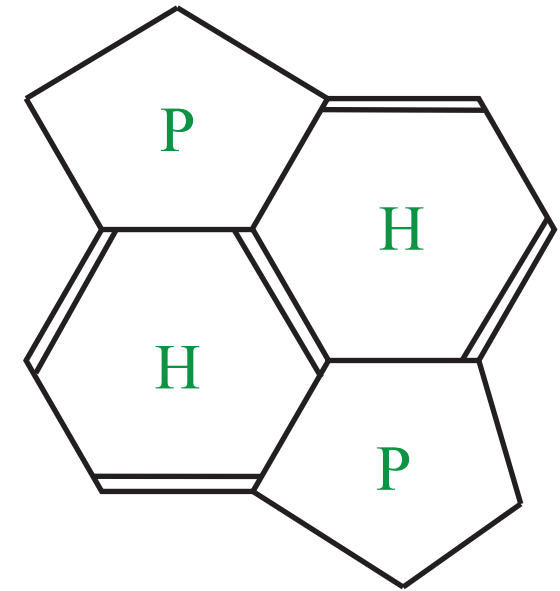
Formula: A_3C_{60}

C_{60} molecule: “soccer-ball” pattern, with 20 hexagons and 12 pentagons: icosahedral symmetry.

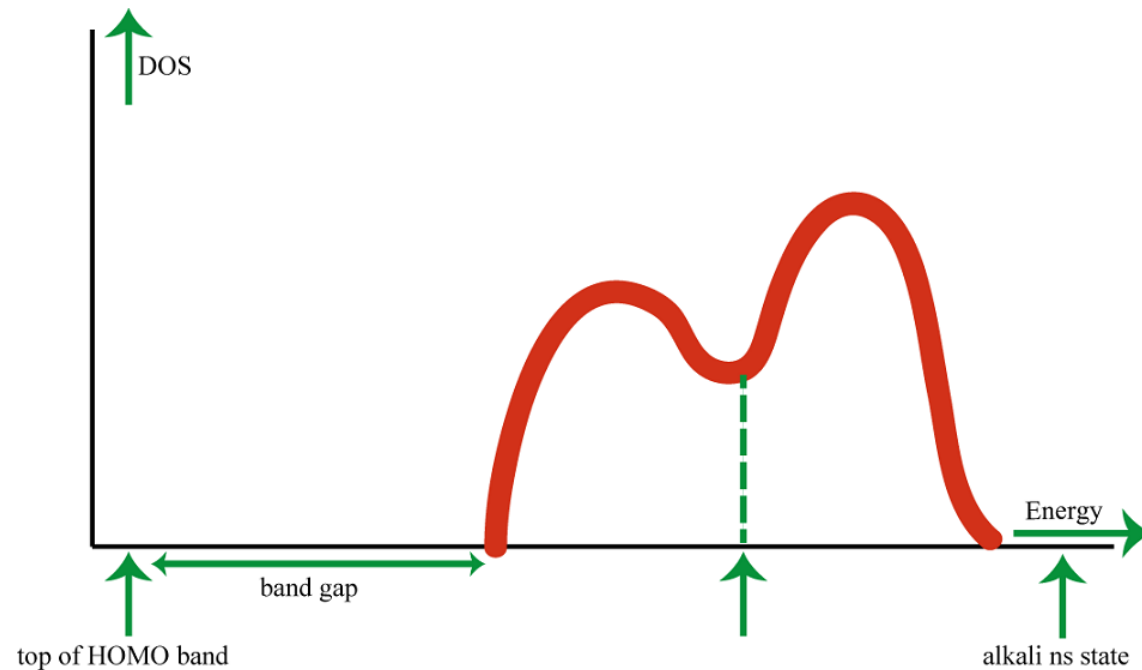
C atom is $1s^2 2s^2 2p^4$, but in molecule or solid, $2s$ and $2p$ hybridize to form 4 sp^2 states. Of these, 3 are used up in the in-plane bonding, leaving 1 electron per C atom (= 60 per fullerene molecule) in p_z -like state (i.e. “sticking out of surface”) (cf. graphene)

The HOMO state is 5-fold orbitally degenerate, the LUMO state 3-fold degenerate.
 “highest occupied molecular orbital” “lowest unoccupied molecular orbital”

In the single molecule, HOMO-LUMO splitting $\sim 0.6\text{eV}$.



Fullerene crystals¹: fcc, cubic lattice parameter $\sim 14.2\text{\AA}$ (corresponds to close-packed molecules)



Bond which evolves from LUMO state has density of states (DOS) roughly like above;
Density of states in middle of band very sensitive to lattice parameter.

¹ignore “merohedral” disorder.

Alkali fullerenes: Any intercalated alkali atoms will **donate their s -electrons** to the LUMO bond, thereafter “spectators” (cf. “charge reservoir” atoms in cuprates).

Normal-state properties roughly consistent with “textbook” picture of half-filled band. ($C_v \sim T$, $\chi \sim \text{const.}$, $T_1 \sim T^{-1}$ etc). Note experiment confirms $N(0) (\equiv \frac{1}{2}(\frac{dn}{d\epsilon}))$ strongly dependent² on lattice constant a . (photoemission, plasmons)

²important for transport, in N and Superconducting states

Alkali fullerenes: superconducting state

Superconductivity occurs, in A_xC_{60} , only very close to $x = 3$, but then has T_c up to $\sim 40\text{K}$ (Cs_3C_{60}) (contrast intercalated graphite, eg KC_8 , $T_c \lesssim 1\text{K}$)

T_c increases with increasing lattice spacing a (due to pressure or substitution): typically,

$$\partial T_c / \partial a \sim 33\text{K}/\text{\AA}$$

Consistent with BCS result

$$T_c \sim \omega_D \exp(-1/N(0)V_0)$$

with V_0 mainly of intramolecular origin, hence independent of a , and density of states $N(0)$ increasing with a .

A_3C_{60} 's strongly type-II, $\underbrace{\xi(0)}_{\text{pair radius}} \sim 26\text{\AA}$ (< 2 lattice spacings), $\underbrace{\lambda(0)}_{\text{London pen. depth}} \sim 2400\text{-}4800\text{\AA}$.

Pairing state:

$\left\{ \begin{array}{l} T_1 + \text{infrared reflectivity} \implies s\text{-state (density of states very small for } E < \Delta \approx 1.76k_B T_c) \\ \text{HS peak seen in } \mu\text{SR and } ^{13}\text{C NMR} \implies s\text{-state (as in "BCS" superconductors)} \end{array} \right.$

($\star : \Delta\lambda(T) \sim T^\alpha, \alpha \sim 3$)

Mechanism:

Isotope exponent ($^{12}\text{C} \rightarrow ^{13}\text{C}$) $\approx 0.4 \implies$ phonon mechanism

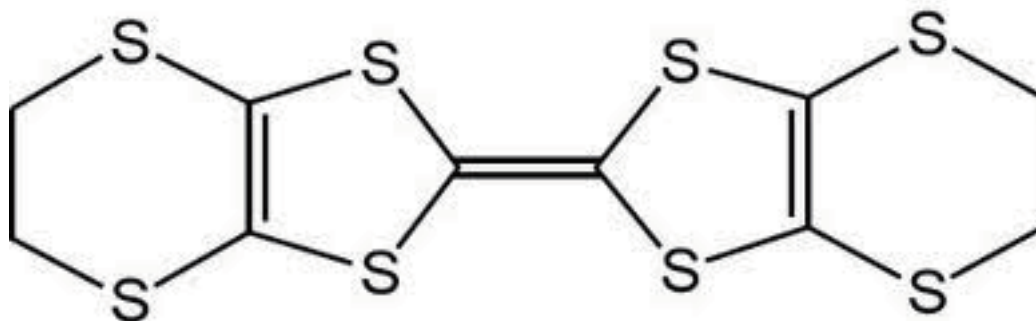
Note: $\omega_D/\epsilon_F \sim 0.3 - 0.6$ (BCS superconductors: $\sim 10^{-2}$)

so even if mechanism is phonon, details may be rather different from BCS.

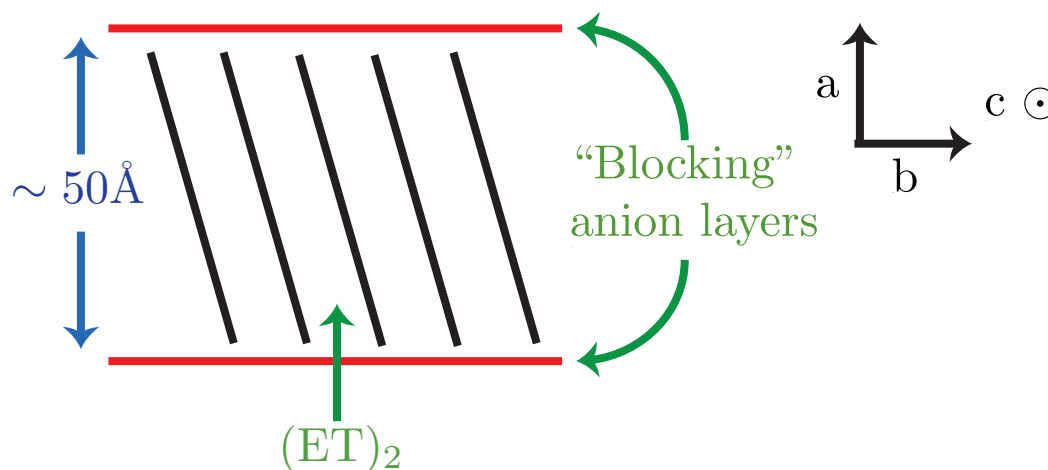
Default explanation:

Alkali fullerenes pretty well described by BCS theory, but strongly molecular structure enables them to avoid usual limit on T_c (cf. MgB_2)

2.Organics



Mostly quasi-2D crystal based on $\text{ET}(\text{BEDT-TTF})$ (bis(ethylene-dithio)-tetrathiafulvalene).
 Structure: $(\text{ET})_2\text{X}$, X =monovalent anion (I_3^- , $\text{Cu}(\text{NCS})_2^- \dots$). Conducting layers are $(\text{ET})_2$, “blocking” layers anions.

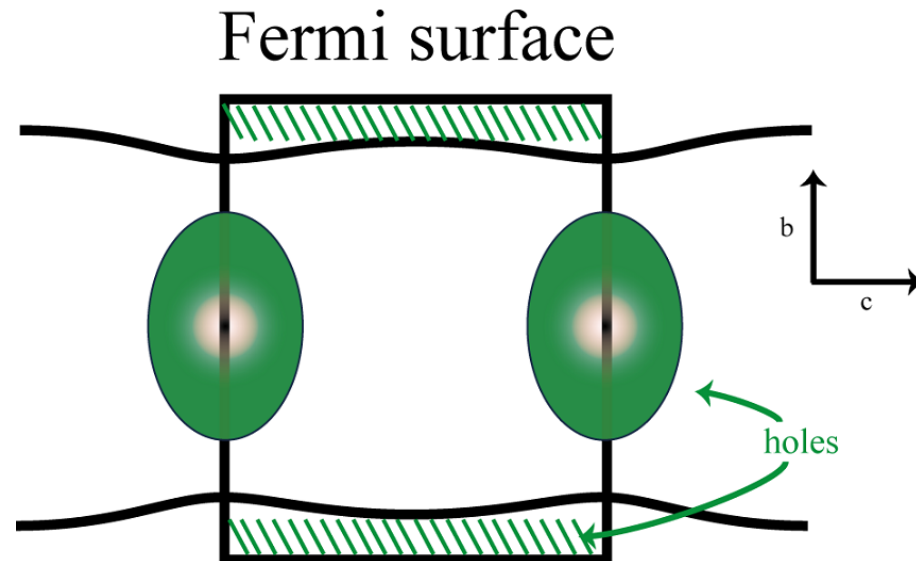


Normal State

Conduction electron density low ($\sim 10^{21}\text{cm}^{-3}$)

Very strong a -axis anisotropy³: $\rho_a/\rho_{bc} \sim 10^2\text{-}10^3$ (comparable to cuprates)

Samples can be made very clean \Rightarrow dHvA-type oscillations observable



Typical Fermi Surface Structure (K-(ET)₂Cu(NCS)₂): very strongly 2-dimensional

³Confusingly, the conventional notation calls the “hard” direction a (not c as in the Cuprates).

Superconducting State

T_c typically $\sim 10\text{-}12\text{K}$ (high in terms of calculated in-plane hopping matrix element)

extreme type-II ($H_{c1} \sim \text{a few mT}$, $H_{c2} \sim 8\text{-}15\text{T}$)

estimated $\xi_{\parallel} \sim 50\text{\AA}$, $\xi_{\perp} \sim 5\text{\AA}$ (\ll interlayer distance)

Symmetry: $c_v(T) \sim e^{-\Delta/T}$ for $T \rightarrow 0 \implies s\text{-wave}$

★: $T_1^{-1} \sim T^3$, no HS peak

Isotope effect: $^{12}\text{C} \rightarrow ^{13}\text{C}$ has $\alpha \sim 0.1$, but $^1\text{H} \rightarrow ^2\text{D}$ produces **inverse** isotope effect ($\alpha < 0$) (effect of lattice deformation?)

Default option: organics somewhat BCS-like, but electronic effects may be competitive.

3.Heavy Fermions⁴

Oldest class of exotic superconductors (1975).

Heavy-fermion systems: compounds containing rare-earth element (usually Ce) or actinide (usually U), with electronic specific heat exceeding “textbook” value by $\sim 10^2$ - 10^3 ($\implies m^*/m \sim 10^2$ - $10^3 \implies$ “heavy”)

(Note: large effective mass seen also in eg. dHvA.) All 3D (not layered).

Normal-state behavior:

at $T \sim 300\text{K}$, behavior of HF systems quite different from textbook metal + not always universal in class (eg. $R(T)$ metallic for UPt_3 , semiconducting for most others). However, generally

$$\left\{ \begin{array}{l} \chi \propto 1/T \\ T_1^{-1} \propto \text{const.} \\ C_V \propto \text{const.} \\ \text{neutron scattering simple Lorentzian peak centered at } T=0 \end{array} \right.$$

\implies consistent with model in which f -electrons ($\text{Ce}^{3+} : 4f_1, \text{U}^{4+} : 5f_2$) form local moments

⁴Ref.: Y.Kuramoto, Y.Kitaoka, Dynamics of Heavy Electrons, Oxford University Press, 2000.

As T lowered, crossover to a Fermi-liquid-like regime:

$$C_V \propto T$$

$$T_1^{-1} \propto T$$

$$\rho \propto A + BT^2 \longleftarrow \text{impurity} + e - e \text{ Umklapp}$$

But coefficient γ in $C_V = \gamma T$ is enormous, up to $\sim 1600 \text{ mJ/mole K}^2$ (CeCu_6) (contrast “textbook” metal, $C_V \sim \text{a few mJ/mole}$).

★: are we sure this specific heat is due to **mobile** electrons?

Yes, since $\Delta c_{n-s}/c_n(T_c) \sim \text{BCS value}$ (+ superconducting electrons presumably must be mobile!)

In Fermi liquid theory, $\gamma \sim m^*$, so $m^*/m \sim 10^2\text{-}10^3$: confirmed by χ (also $\sim 10^2\text{-}10^3 \times$ textbook value), dHvA.

Naive picture of N state:

f -electrons form very narrow band, width $\sim \Delta \sim \text{a few K}$. Then for $k_B T \ll \Delta$, can equally well represent in terms of states **localized** on lattice sites.

$\Rightarrow \chi \propto 1/T, C_V$ “small”, $T_1^{-1} \propto \text{const.}$, etc. For $k_B T \lesssim \Delta$ need proper “band” picture with large $m^*(\propto \Delta^{-1})$.

★: ignores conduction (s or d) electrons! better picture involves competition between Kondo effect (favors s - f singlet) and RKKY (Ruderman-Kittel-Kasuya-Yosida interaction) (favors magnetic ordering of f -electrons). In fact, many HF systems (even some superconducting ones) show antiferromagnetism at $T \lesssim 20\text{K}$.

Heavy Fermion superconductors

4 classes of HF systems:

- 1) no phase transition (ex.: CeAl_3)
- 2) magnetic transition only (ex.: CeCu_6)
- 3) superconducting transition only (ex.: UPt_3 , CeCu_2Si_2 , UBe_{13} ...)
- 4) magnetic and superconducting transitions (ex.: UPdAl_3 , URu_2Si_2 , UGe_2 ...). In this class, magnetic order and superconductivity **coexist** (contrary to established “text-book” wisdom!)

In all cases of superconductivity in a HF system, $T_c \lesssim 2\text{K}$. (In class 4, $T_N \sim 10\text{-}50\text{K}$).

Diagnostics: most crucial observation is that

no HFS shows any appreciable isotope effect

\implies strongly suggests non-phonon (“all-electronic”) mechanism.

Pairing state:

Need to discuss each HFS separately: diagnostics include low-T behavior of C_V , T_1^{-1} , κ_{el} , Knight shift, H_{c2} , sensitivity to nonmagnetic scattering, multiple phases...

Some representative HFS and their (suggested) symmetries⁵

System	Magnetic?	T_c (K)	Parity	Gap nodes?	Comments
UPt ₃	P	0.56	—	✓	
CeCu ₂ Si ₂	P	0.65	+(?)	✓	
UBe ₁₃	P	0.9	—(?)	✓	
UPdAl ₃	AF	2.0	+	✓	$T_N = 14.5K$
CeCoIn ₅	P	2.3	+	✓	probably $d_{x^2-y^2}$
UNiAl ₃	AF	1.0	—	?	$T_N = 4.6K$
URu ₂ Si ₂	AF	0.8	—	?	$T_N = 17.5K$
UGe ₂	F	0.6	—	?	$T_{Curie} = 30K^*$

* at point of maximum T_c .

⁵Note: no evidence for T-violation in any HFS.

4. Strontium Ruthenate : Sr_2RuO_4

History

Superconductivity in cuprates up to ~ 150 K

Typical (original) cuprate : $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$ ($T_c \sim 40\text{K}$)



Quasi -2D CuO_2 planes appear to be essential to high- T_c superconductivity.

How essential is the Cu? Try replacing it : Ag, Au, doesn't work, but :

Cu ($Z=29$) : $[\text{Ar}] + 3d^{10}4s^1 \rightarrow 3d^9$

Ru ($Z=44$) : $[\text{Kr}] + 4d^75s^1 \rightarrow 4d^4$



Normal-state properties quite dissimilar
 $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$ has ($T_c \sim 40\text{K}$)
 Sr_2RuO_4 has T_c of ?

Experimental properties* of Sr_2RuO_4

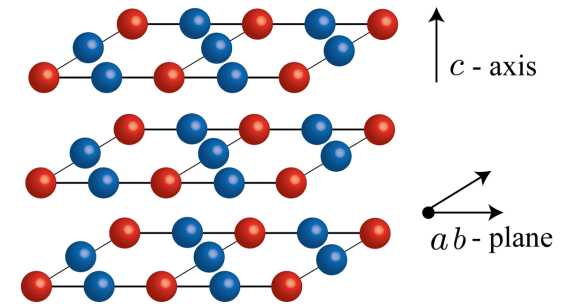
Normal Phase

Below $\sim 25\text{K}$, appears to behave as strongly anisotropic Fermi liquid
(nb : cuprates quite different)

$$C_V \sim \gamma T + \beta T^3 \quad \chi \sim \text{const.}$$

$\rho \sim A + BT^2$ both in ab -plane and along c -axis
(characteristic of coherent (Bloch) wave transport limited by e^- - e^- Umklapp scattering). ρ_{ab} small
($\sim 1\mu\Omega \text{ cm}$)

\implies samples very pure.



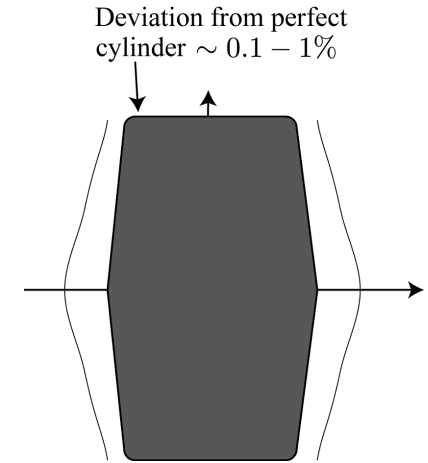
* Mackenzie, and Maeno, RMP **75**, 1 (2003)

However, $\rho_c/\rho_{ab} \sim 10^3$ (comparable to cuprates)

Band structure:

Experiment (dHvA, Shubnikov-De Haas) and theory (LDA) agree:

Fermi surface consists of 3 strongly 2D sheets: α (hole-like), β , γ (electron-like)



	α	β	γ
$k_F(\text{\AA}^{-1})$	0.3	0.6	0.75
m^*/m_e	3.3	7.0	16.0
m^*/m_{band}	3.0	3.5	5.5

indication of
strong correlations

$\chi \sim \text{const.}$ in superconducting state \Rightarrow triplet equal-spin-pairing

Suppose order parameter of Sr_2RuO_4 is indeed of equal-spin-pairing form:
then

$$F(\mathbf{k}; \sigma_1, \sigma_2) = F(\mathbf{k}; \sigma_1, \sigma_2) = f(\mathbf{k})(\uparrow\uparrow + \downarrow\downarrow)$$

Then the crucial question is :

What is $f(\mathbf{k})$? \leftarrow orbital wave function of pairs

$f(\mathbf{k})$ odd parity (consistent with Josephson experiments)

In particular, is it real (e.g. $f(\mathbf{k}) \sim k_x$) or complex (e.g. $f(\mathbf{k}) = k_x + ik_y$)?

\uparrow

breaks T-invariance

In BCS theory, want $|\text{OP}|^2$ to be as uniform as possible over Fermi surface $\rightarrow k_x + ik_y$ always favored. But in more general theory, need not be so. Experiments favoring violation of T-symmetry:

a) Muon spin rotation extra "internal" magnetic field in superconductor state (\uparrow : apparently H has ab -plane component!)

b) Magnetic field dependence and telegraph noise in I_c of Josephson junctions interpreted in terms of switching of domains ($p_x + ip_y \rightleftharpoons p_x - ip_y$)

c) Kerr effect in zero magnetic field.

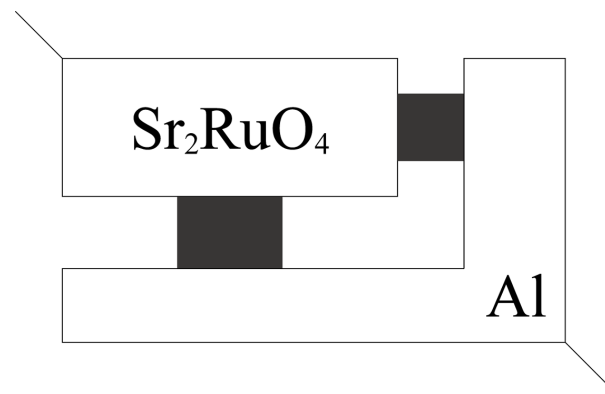
★ : "ideal" $p_x + ip_y$ state ($\Delta \sim F \sim \text{const.}(k_x + ik_y)$ has no nodes)

\implies exponentially small no. of quasiparticles for $T \ll T_c$

\implies no appreciable specific heat, thermal conductivity.....

In fact, experimental evidence for power law contribution of many of these quantities \rightarrow gap has nodes?

In principle, critical test I_c max. at $\Phi = \frac{1}{4}$ or $\frac{3}{4}\phi_0$



5. Ferropnictides

★: only 3 years old, so much experimental data may not be definitive.

Composition: two major atoms, each containing a transition metal (usually Fe) and a pnictide (element in N column of periodic table), usually As.

2 main classes of parent compounds:

1111: $\underbrace{(\text{RE})}_{\text{rare earth}} \underbrace{(\text{TM})}_{\text{transition metal}} \underbrace{(\text{PN})}_{\text{pnictide}} \text{O}$: example, $\text{LaFeAsO}_{(1-x)\text{F}_x}$

122: $\underbrace{(\text{AE})}_{\text{alkaline earth}} (\text{TM})_2 (\text{PN})_2$: example, BaFe_2As_2

third class: LiFeAs , FeSe ... ← “11”

most work on (1111), will mostly refer to this.

Structure (1111 compounds) (schematic)

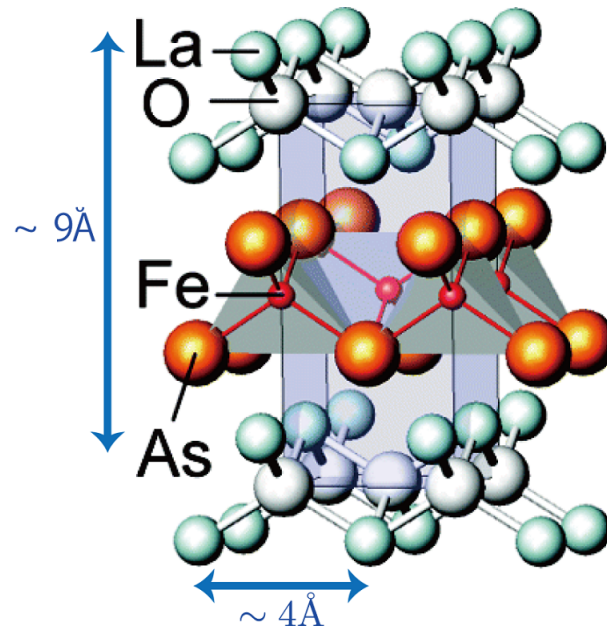


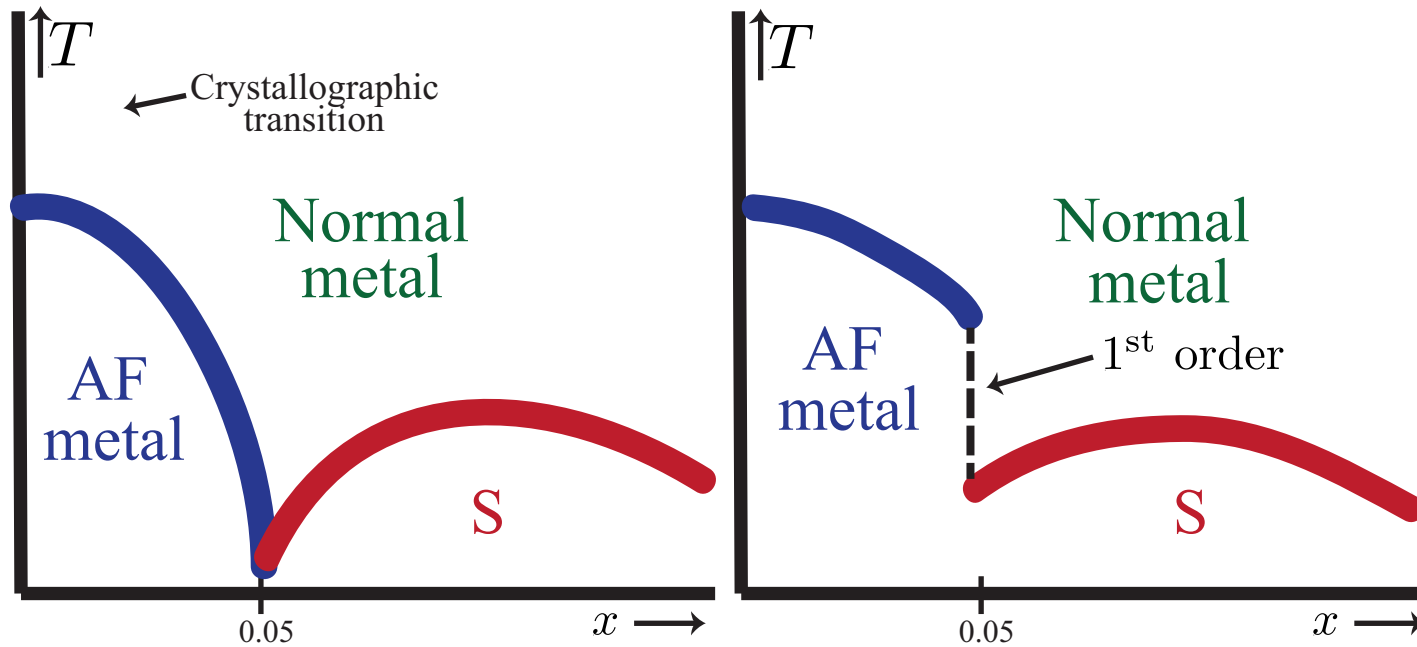
Figure 1: Kamihara et al., J.Am.Chem.Soc., 2008, 130 (11)

in parent compound, valence state probably $(\text{La}^{3+}\text{O}^{2-})^+(\text{Fe}^{2+}\text{As}^{3-})^-$ (+ some Fe 3d-As 4p hybridization)
 (doubly) closed shell \uparrow $\uparrow 3d^6$ \uparrow closed shell

If F substituted for O, extra electron \rightarrow FeAs layer: for O_{1-x}F_x , $x \sim 0.1$, carrier density $\sim 10^{21} \text{ cm}^{-3}$
 (comparable to cuprates)

Note that as in cuprates, “charge reservoir” (F’s in LaO layer) well separated from metallic (FeAs) layer. Note: 1111’s are electron-doped (but 122’s hole-doped)

Phase diagram



Note max. in $T_c(x)$ at $x \sim 0.12-0.15$ is very “shallow” compared to cuprates

Experimental properties (N state)

$$\left\{ \begin{array}{ll} C_v & \sim \alpha T + \beta T^3 \\ \chi & \sim A + BT \\ \rho \text{ (dc conductivity)} & \sim A + BT^2, \sim 3\text{m}\Omega \text{ at RT } (\sim \text{cuprates}) \\ \text{Hall coefficient} & \sim A + BT \end{array} \right.$$

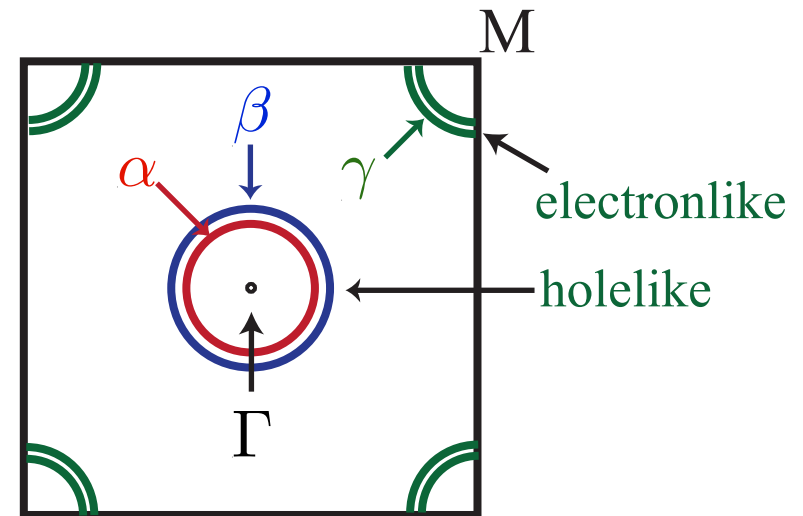
★: so far, anisotropy not measured (?)

overall fairly “textbook”

Band structure:

$$\left\{ \begin{array}{l} \text{ARPES} \\ \text{quantum oscillation} \\ \text{LDA calculations} \end{array} \right. \Rightarrow$$

(★ beware notation!)

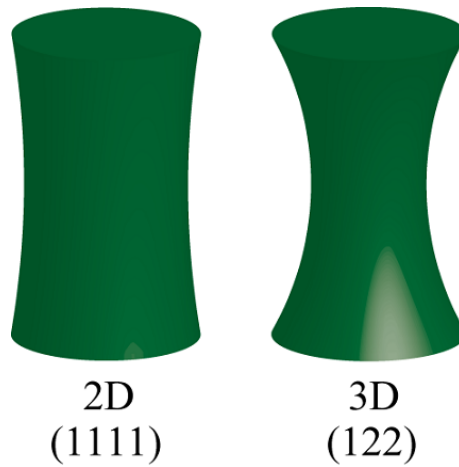


Note:

in 1111 Fermi surface very 2D. (“barrel-like”)

in 122 much more 3D.

also, magnetism much stronger in 122.



Superconductivity

T_c up to $\sim 56\text{K}$ in (doped) 1111, 38K in 122, 20K in 11. : weakly x -dependent (e.g. $\text{LaFeAsO}_{1-x}\text{F}_x$): a surprise, because Fe ions have magnetic moments $\sim \mu_B$ (neutron scattering).

Experimental properties (Superconducting state)

All ferropnictides strongly type-II, with (extrapolated) $H_{c2}(0) \sim 55\text{T}$:

(exceeds Chandrasekhar Clogston limit) \uparrow

anisotropy relatively small (~ 2 -3). $\lambda_{ab}(0) \sim 1600$ -2400Å, $\xi_{ab}(0) \sim 20\text{Å}$
ARPES: on all sheets, gap only weakly \mathbf{k} -dependent.

NMR, penetration depth: at low T data mutually inconsistent, but favor power law.

very importantly: Knight shift $\rightarrow 0$ for $T \rightarrow 0$ for **all** directions of magnetic field.

Isotope effect: experiments mutually inconsistent, but most recent give small value of α

\implies suggests mechanism mostly non-phonon. (consistent with firm theoretical prediction that phonon mechanism cannot give 55K)

The pairing state

Knight shift \implies spin state singlet (fairly firm conclusion)

For (approximate) tetragonal symmetry, main candidates are s (presumably nodeless) and $d_{x^2-y^2}$ (nodes at (π, π)).

ARPES data suggest nodeless $\implies s$. but $T \rightarrow 0$ T_1, λ rather favor nodes

★: However, extra complication:

gap need not have same sign on all sheets of Fermi surface!

Theory based on spin-fluctuation mechanism (most “obvious” non-phononic mechanism) predicts gap changes sign between electronlike and holelike sheets (“ s^\pm ”)

Experimental evidence claimed in favor of this assignment:

“Josephson-like” experiment on polycrystalline sample⁶ (shows half flux quanta)

STM⁷

No “true” Josephson experiments yet...

⁶Chen et al., Nature Physics **6**, 260 (2010)

⁷Hanaguri et al., Science **327**, 474 (2010)