東京大学 THE UNIVERSITY OF TOKYO

November 11, 2005
University Lecture
Science of Material

Lecture 6 What is Condensed Matter Physics?

Lecture 7 Quantum Mechanics and Artificial Materials

-- High-tech and the State-of-the-art Physics

Lecture 8 Atom Control and Quantum Control

--Nano-science and Quantum Information

Lecture 9 Diverse Matter and Physical Properties

The University of Tokyo,
The Institute For Solid State Physics

Yasuhiro Iye

Review of Lecture 6 (1)

- Story about scale.
- Modern civilization and physics.
  - Personal computers, cellular phones, satellite communications, GPS, and MRI.
  - Semiconductors, magnetic bodies, dielectric, liquid crystals, gels, superconductors.
- Condensed matter physics is the field of physics.
  - Matter and materials.
  - Materials science and materials engineering.
  - Incorporation of the concept with elementary particle physics.
  - The hierarchy structure in the physical world. ⇒ emergent properties.
  - Diversity in matter and physical properties.

Review of Lecture 6 (2)

- Quantum mechanics and atomic structure.
  - Energy level in atoms ⇒ shell structure ⇒ periodic law.
  - The energy scale ~eV in the outermost shell is the most essential to the properties of the element.
- The existing forms of matter.
  - Energy versus entropy ⇒ phase transition.
- Agglutination mechanism and crystal structure in atom.
  - Crystal structure ⇒ X-ray（electron beam, neutron beam) diffraction.
  - The bonding force of atoms: Van der Waal bonding, ion bonding, covalent bonding, metal bonding, hydrogen bonding.
- ⇒ Diversity of matter and physical properties. (Lecture 9)

Today’s Topics

- Overview of quantum mechanics
- Quantum interference
- Artificial materials in mesoscopic systems
- Quantum conduction

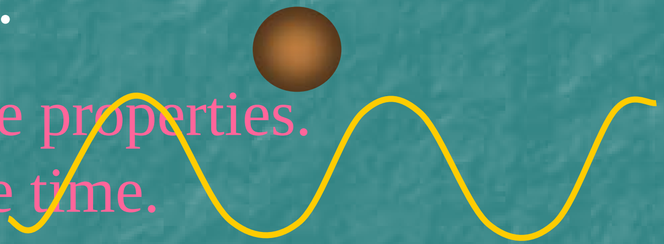
Overview of Quantum Mechanics

Quantum Mechanics

Theoretical description of behaviors in microscopic systems.

Structure of atoms and molecules.
The behavior of an electron in a solid state.
Visible radiation and matter.

Light: possesses both wave and particle properties.
Electron: particle and wave at the same time.



The expressions such as "particle" and "wave" used to refer to everyday phenomena (classical mechanics) is now considered just an analogy that contributes to the phenomena relevant to quantum mechanics.

Particle properties: discreteness and separatenes
Wave properties: superposition and interference

In Quantum Mechanics, a Particle also Behaves as a Wave

de Broglie wavelength: $\lambda = \dfrac{h}{p}$

Momentum

de Broglie wavelength of electron accelerated at 100V:

$$E = \frac{p^2}{2m} \quad \Rightarrow \quad p = \sqrt{2mE}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$= \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 0.91 \times 10^{-30} \times 100 \times 1.6 \times 10^{-19}}}$$

$$= 1.23 \times 10^{-10}\,\text{m} \qquad \lambda = 0.12\,\text{nm}$$

de Broglie wavelength in classical particles, e.g., tennis ball is extremely short.

Quantum Mechanics

The state of a particle is described by a wave functic $\psi(x,y,z,t)$

The existence probability of a particle is given by $\left|\psi(x,y,z,t)\right|^2$

Time evolution of a wave function follows Schrodinger's equat

$$i\hbar\frac{d}{dt}\psi(x,y,z,t)=\left(-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+V(r)\right)\psi(x,y,z,t)$$

Linear equation $\Rightarrow$ superposition principle

$\psi_1,\quad \psi_2 \quad\Rightarrow\quad \psi_1+\psi_2$ $\Rightarrow$ Quantum interference

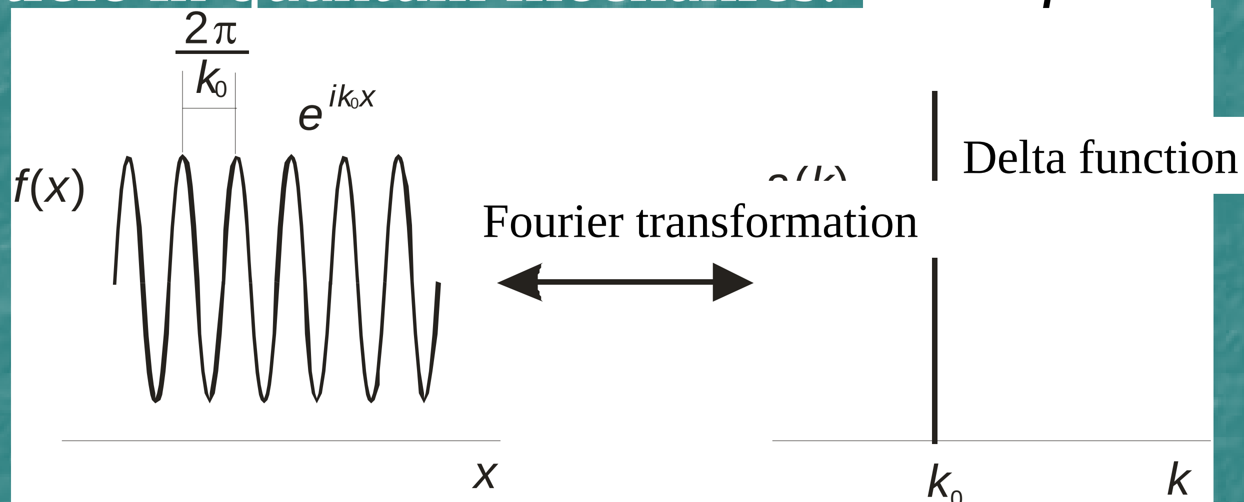
Uncertainty Relation

There is uncertainty relation between position and momentum for particle in quantum mechanics. $\Delta x \cdot \Delta p \geq \hbar$

Defined momentum $p = \hbar k_0$

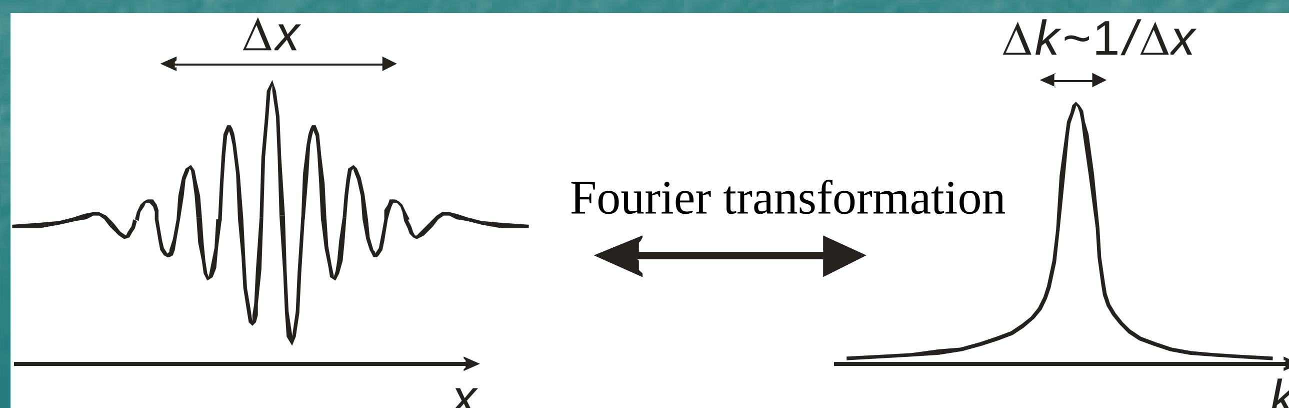
Plane wave

$$\psi(x) = e^{ik_0 x}$$



Wave packet

$$\psi(x) = \sum_k c(k) e^{ikx}$$

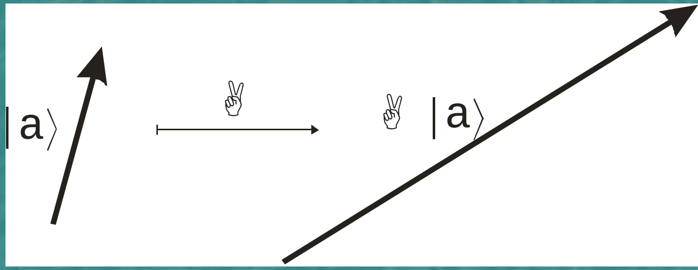


Quantum-mechanical State and Measured Value

The existence probability of a particle can be obtained b $\left|\psi(x,y,z,t)\right|^2$

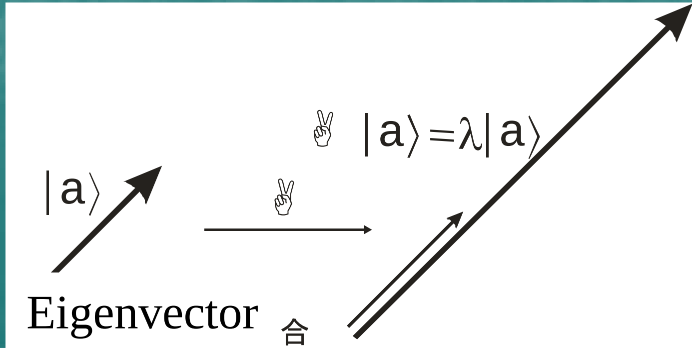
The exact point of existence can be determined by measuring the position of the particle.

The state in quantum mechanics is described in "wave function", in other words "Hilbert space", i.e., state vector in abstract space.



Each physical quantity has corresponding operator:

eigenfunction and eigenvalue of operator.



Linear algebra ✌ $\psi = \lambda\psi$

Quantum Mechanical Measuring

In general, quantum mechanics can give probability distribution of measurements obtained by repeated measurement of the "same state" physical quantity but, it cannot give the probability distribution of measured values of different and individual states.

Measurement determines the quantum state in a single eigenstate of physical quantity ⇒ "contraction of the state"

Time evolution in Schrodinger's equation and the contraction of wave function by measurement.

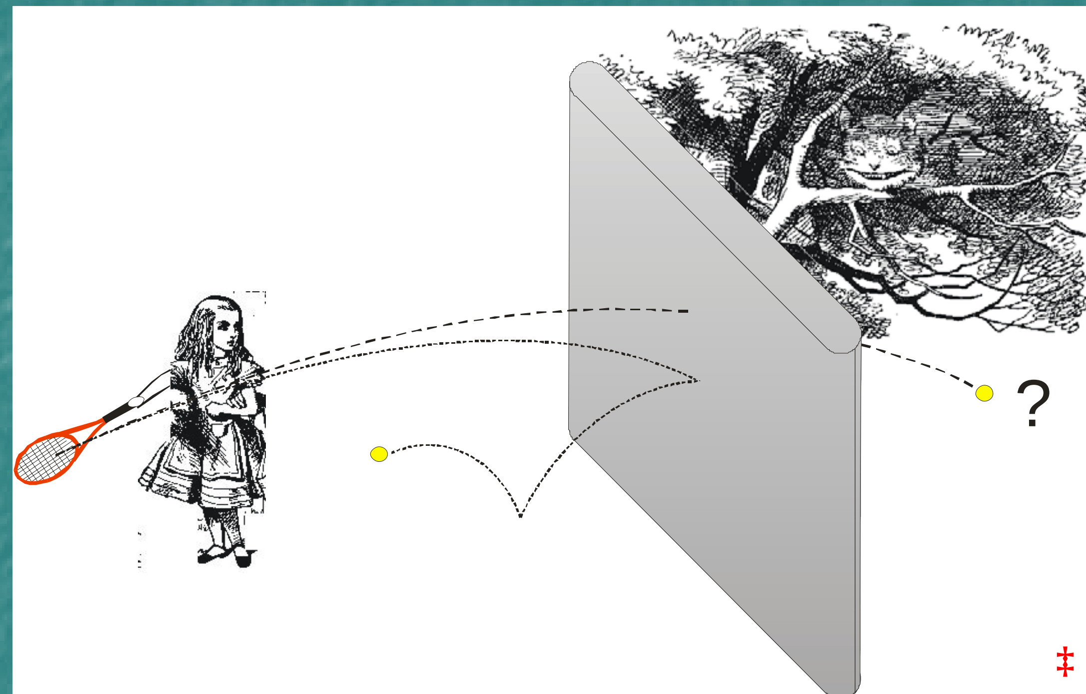
The most common interpretation in quantum mechanics: Copenhagen interpretation.

The observation problem and interpretation problem of quantum mechanics

Characteristic Phenomena in Quantum Mechanics

Tunnel effect

- Particle can pass through a potential barrier, something that is inconceivable in classical mechanics.



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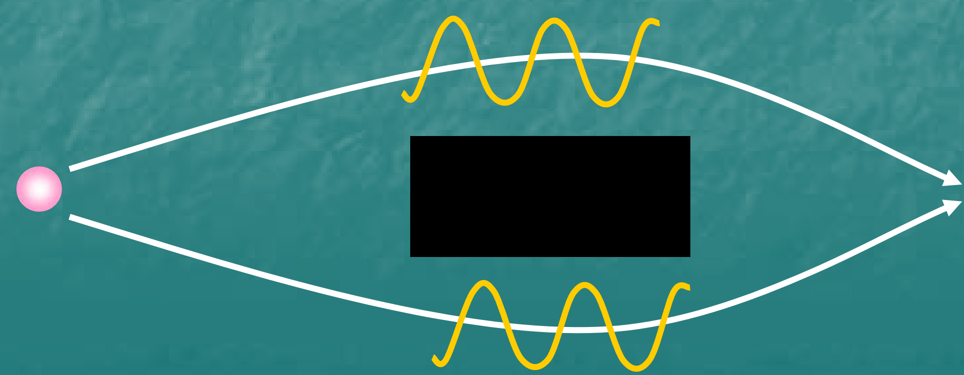
Quantum interference effect

- Superpositioning of different paths. ⇒ quantum interference.



Quantum Interference

Light Wave Interference

The double-slit experiment by Young. (1805)

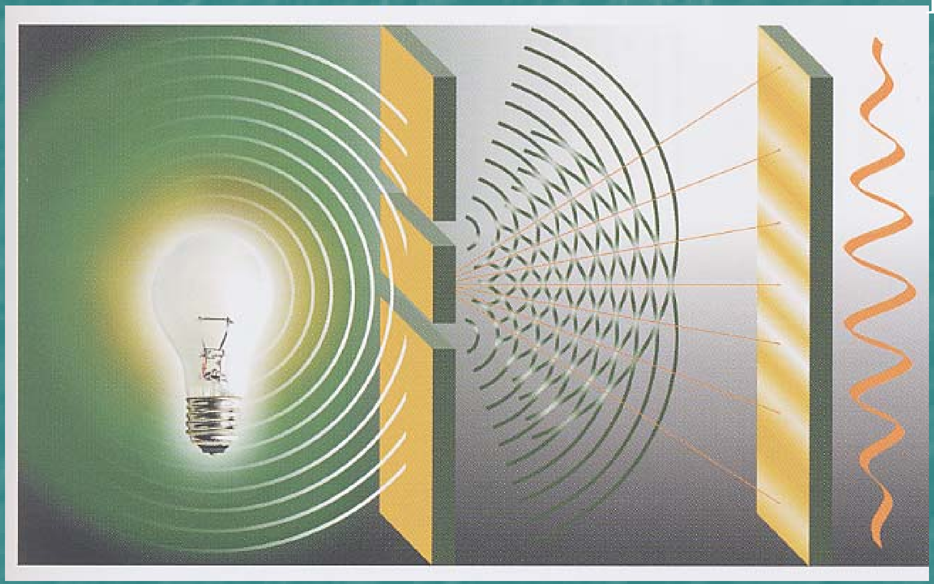


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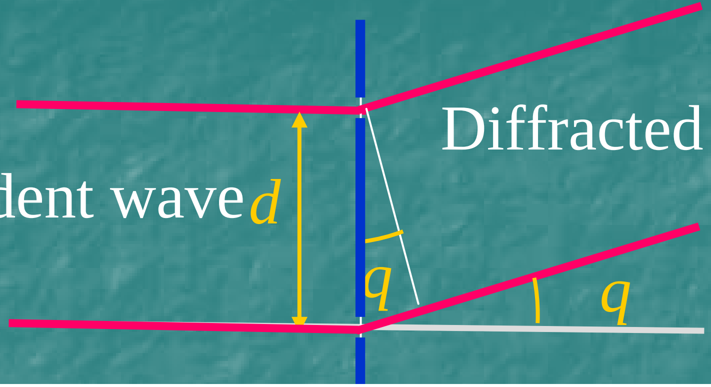
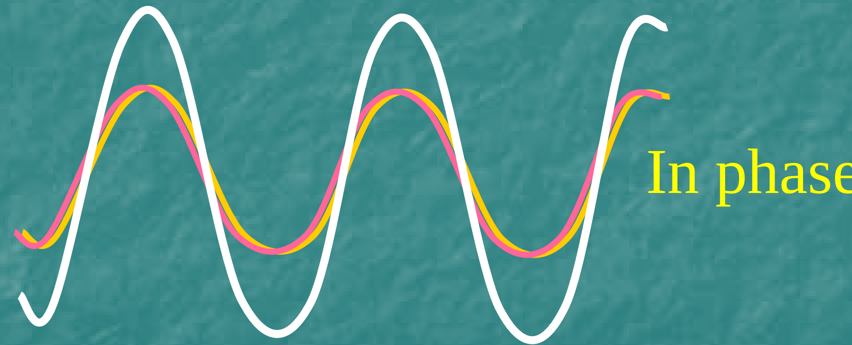
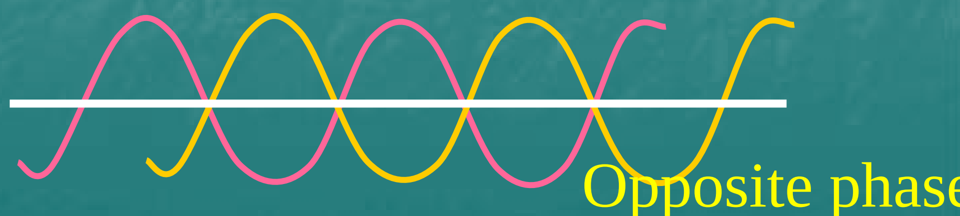
Image of interfering waves as light passes through each slit.

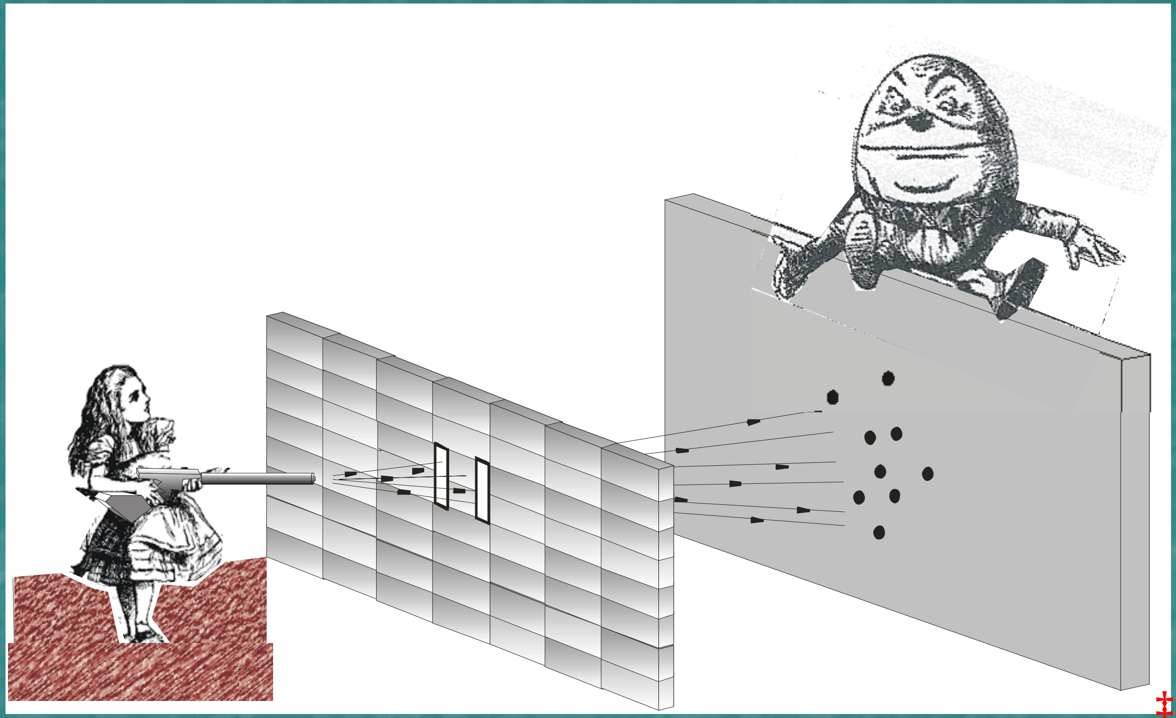


$$d \sin\theta = \begin{cases} n\lambda & \text{Intensifying} \\ \left(n+\frac{1}{2}\right)\lambda & \text{Canceling} \end{cases}$$





For Classical Particles



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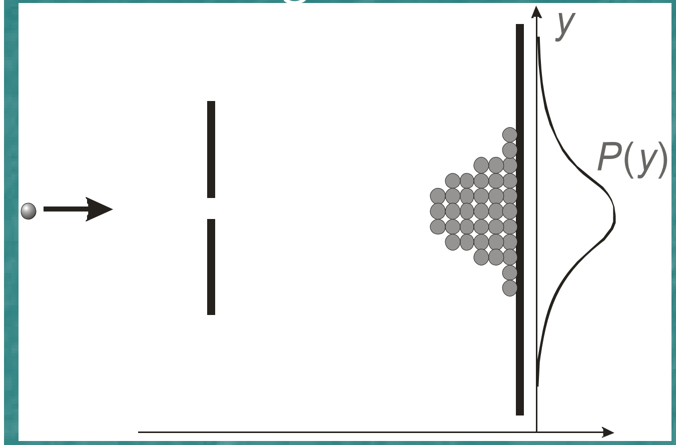
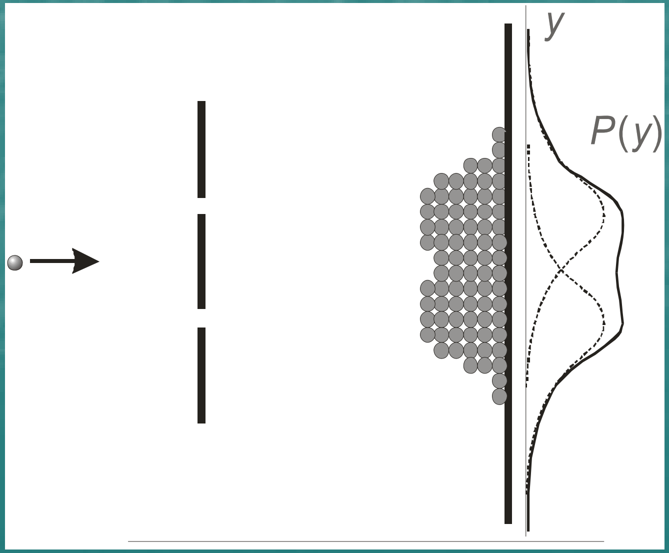
Single slit



A double slit



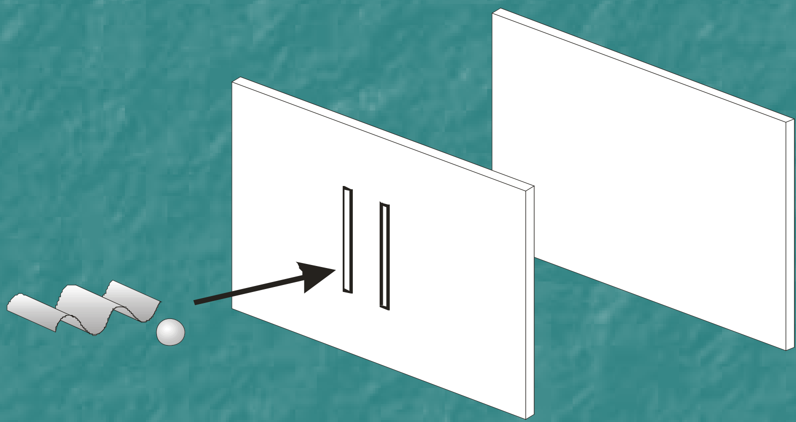
Probability of bullets hitting the wall:

＝probability of bullets hitting the wall through the right slit.

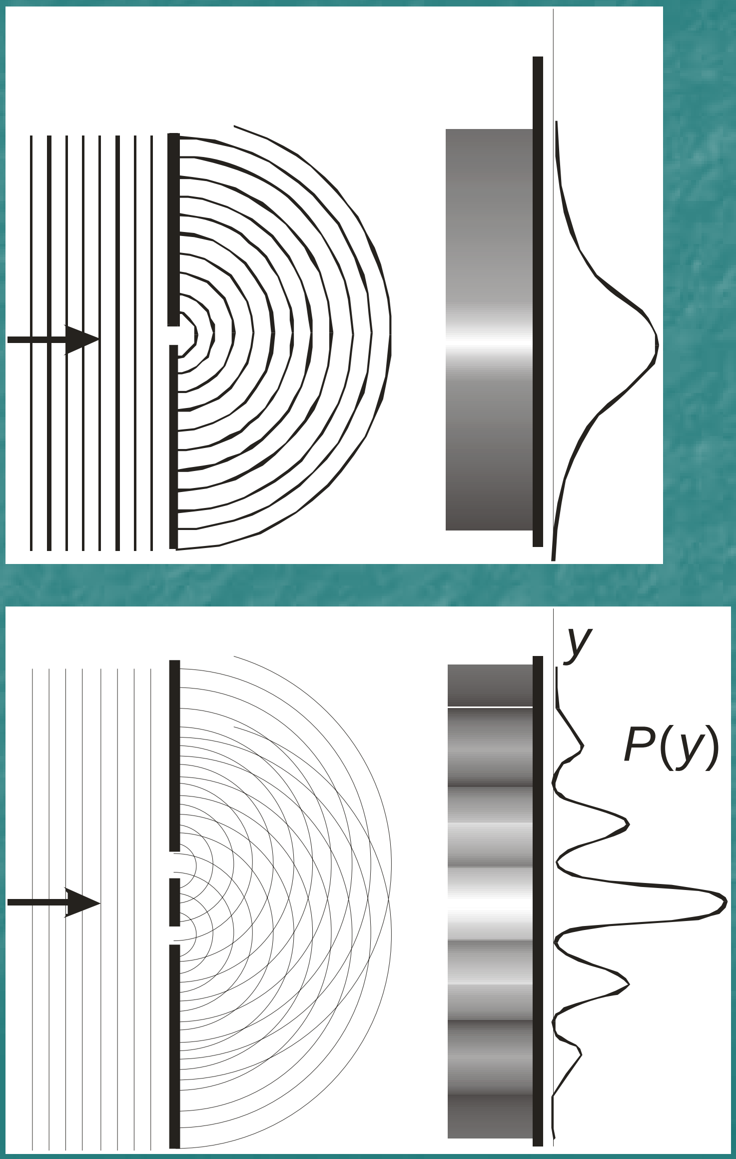
＋probability of bullets hitting the wall through the left slit.

$$P_{\text{total}}(y) = P_{\text{R}}(y) + P_{\text{L}}(y)$$

The Incident Wave



Interference pattern



Particles in Quantum Mechanics

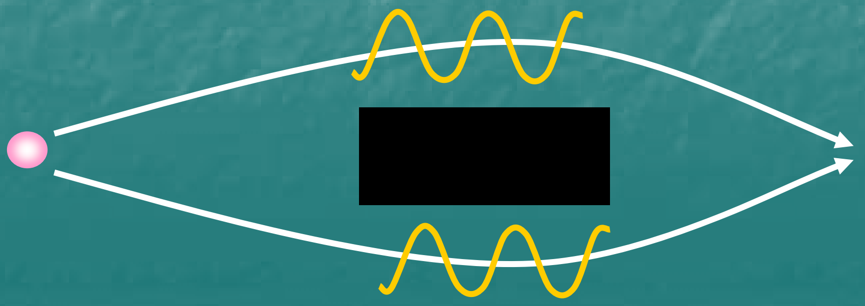
$$\Psi_{\text{total}} = \Psi_R + \Psi_L$$

Wave function＝Wave function for particles passing thorough the right slit.
＋Wave function for particles passing through the left slit.

Probability＝｜Wave function｜^{2}

$$|\Psi_{\text{total}}|^2 = |\Psi_R + \Psi_L|^2$$
$$= |\Psi_R|^2 + |\Psi_L|^2 + \Psi_R^* \Psi_L + \Psi_R \Psi_L^*$$

Quantum interference term



Electron Interference

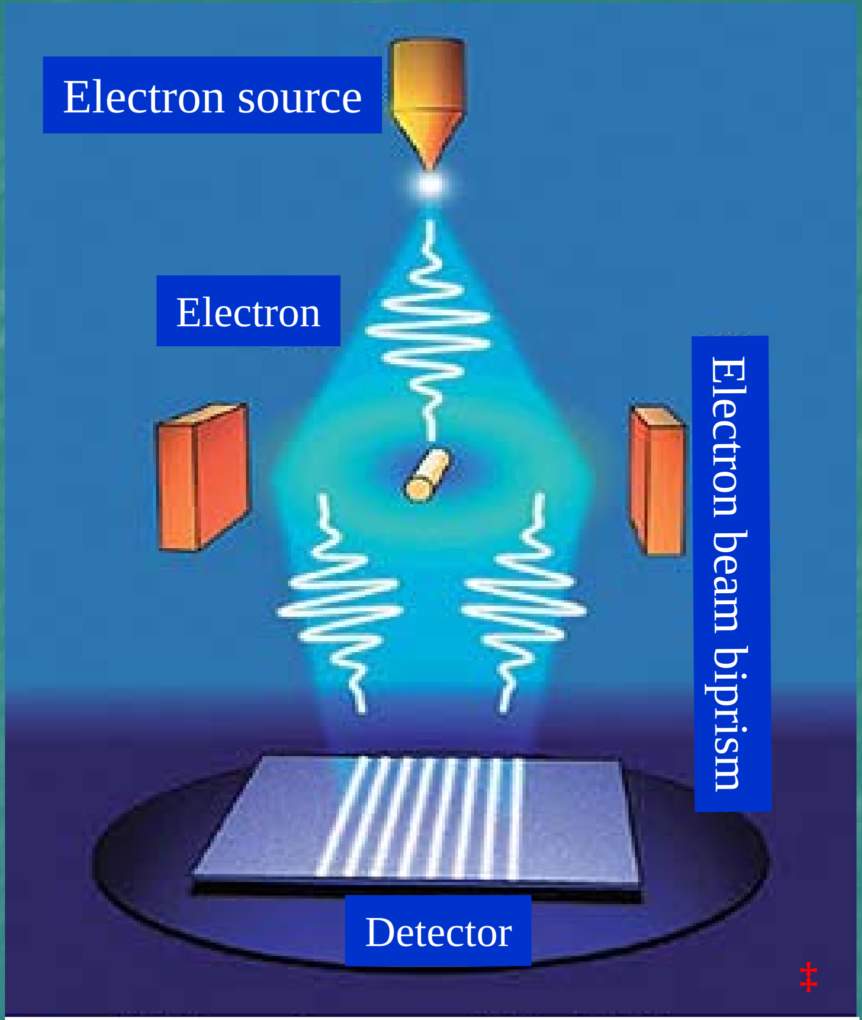
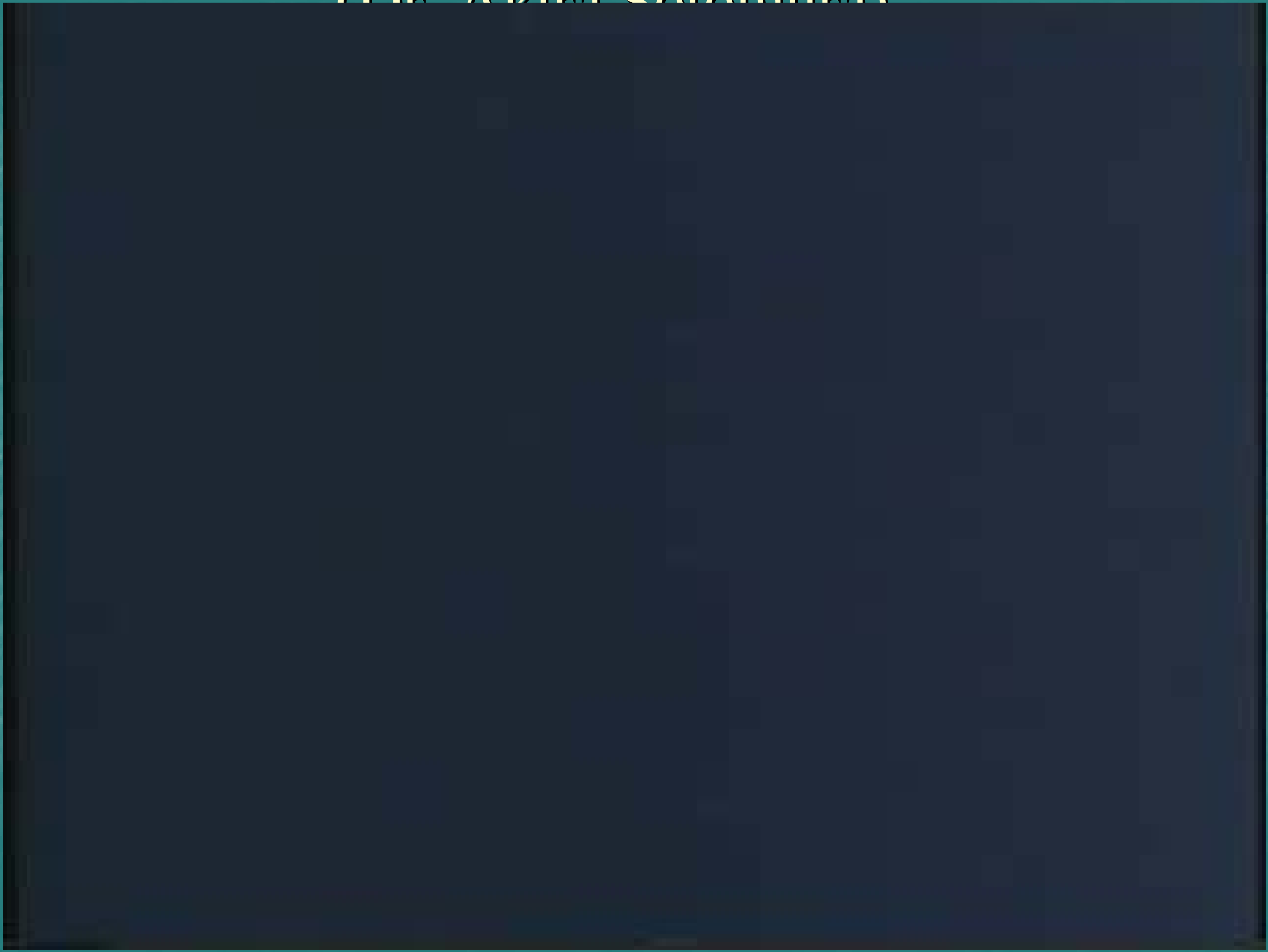


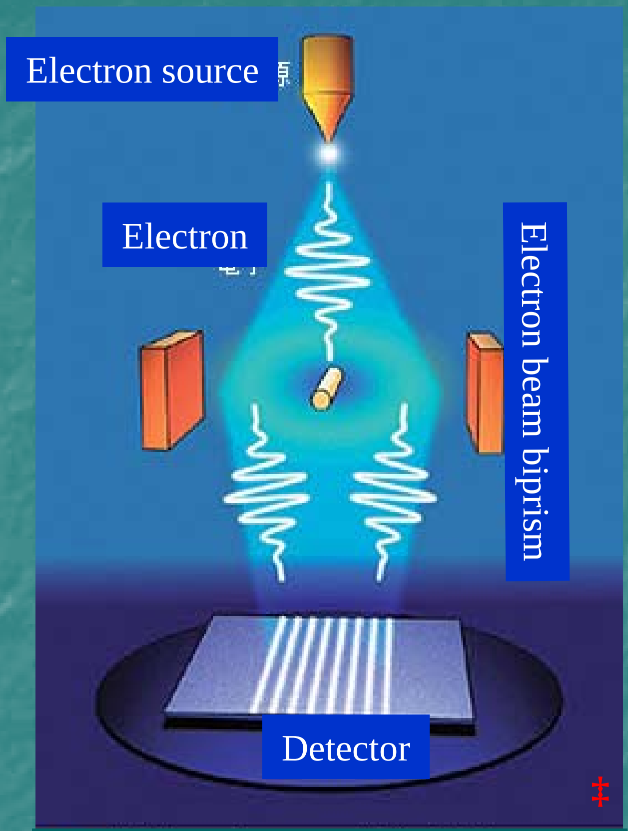
Figure 3. A double-slit experiment of electrons: Electron beam biprisms are used to replace slits. The experiment was so difficult to perform that people used to think that it could be realized only in one’s mind and not in reality.

Dr. Akira Sotomura (Advanced Research Laboratory, Hitachi, Ltd.)

The Double-slit Experiment of Electron (Dr. Akira Setomura)



Electron Interference

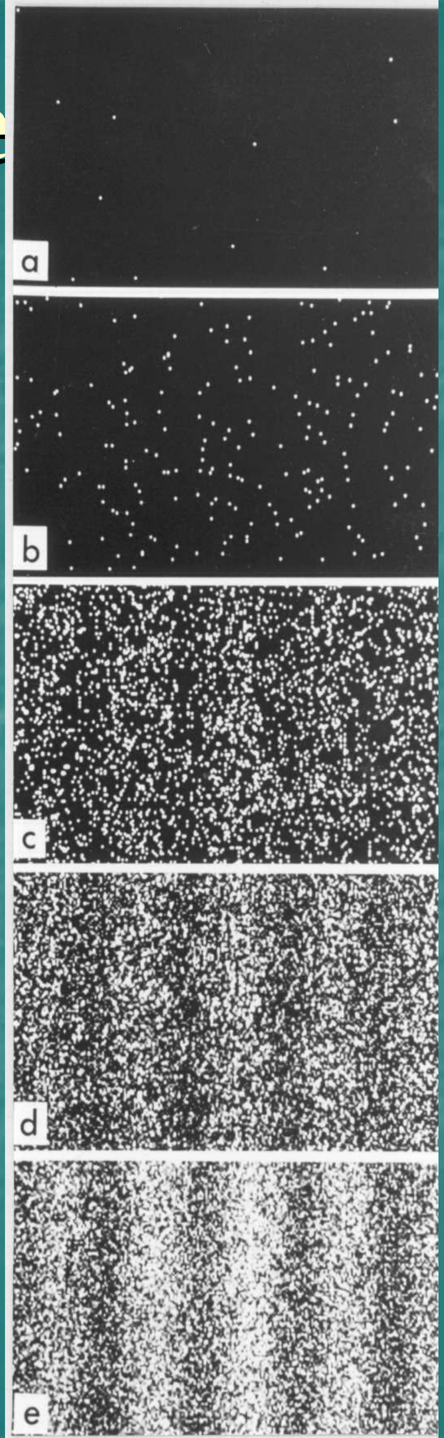


Dr. Akira Sotomura
(Advanced Research Laboratory, Hitachi, Ltd.)

Electrons individually reach the screen.

Interference pattern appears.

Clear evidence for an electron’s wave properties.



Aharonov-Bohm Effect (AB Effect)

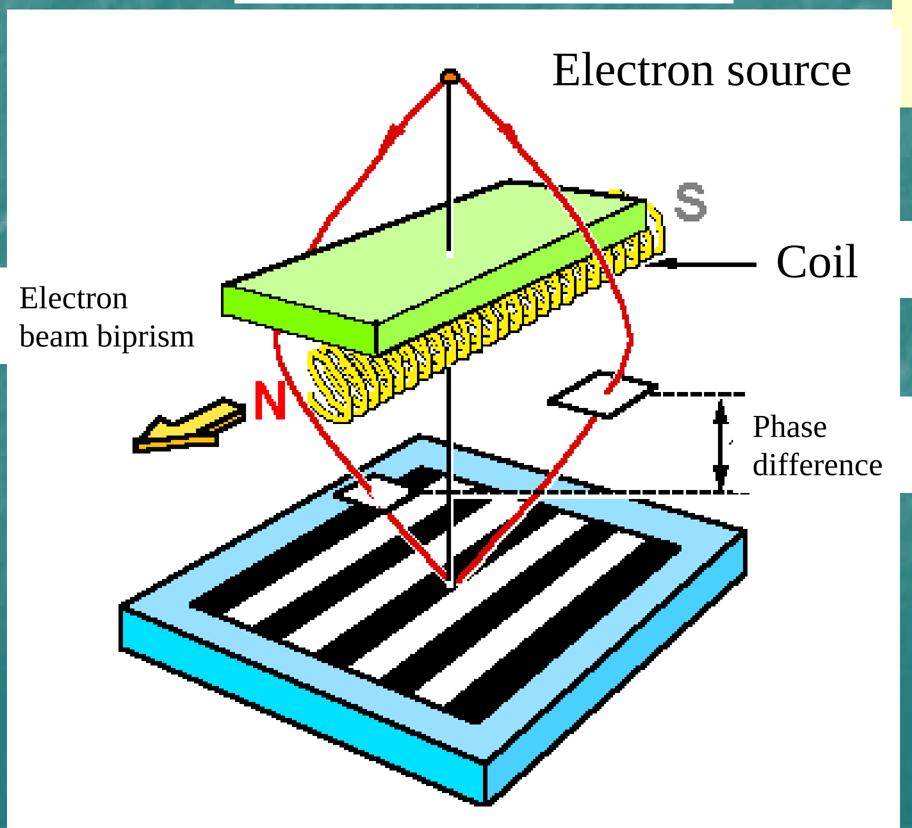
The magnetic field（vector potential）changes the phase of the elec

$$\psi \Rightarrow \psi\, e^{i\frac{e}{\hbar}\int \mathbf{A}(\mathbf{r})\cdot d\mathbf{r}}$$

$$\Delta\theta = \frac{\hbar}{e}\int_{\mathrm{L}} \mathbf{A}(\mathbf{r})\cdot d\mathbf{r} - \frac{\hbar}{e}\int_{\mathrm{R}} \mathbf{A}(\mathbf{r})\cdot d\mathbf{r}$$

$$= \frac{\hbar}{e}\oint_{\mathrm{loop}} \mathbf{A}(\mathbf{r})\cdot d\mathbf{r} = \frac{\hbar}{e}\int \mathbf{B}(\mathbf{r})\cdot d\mathbf{S}$$

$$= 2\pi\frac{\phi}{\phi_0} \qquad \phi_0 = \frac{h}{e}$$



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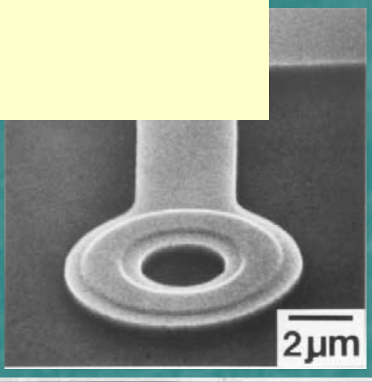
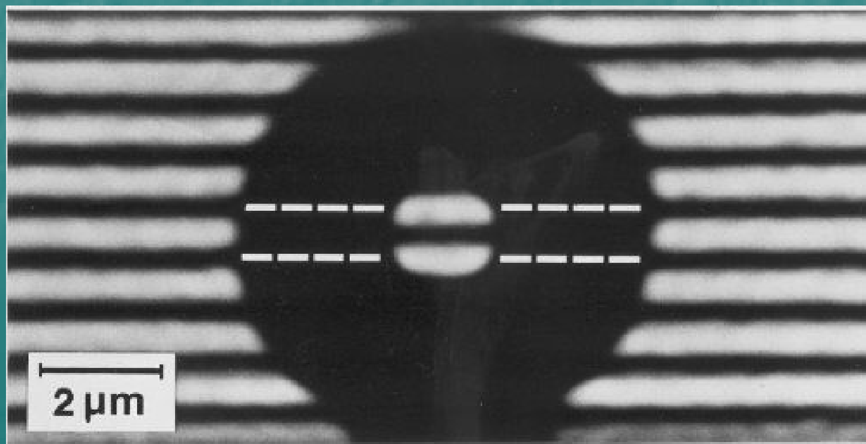
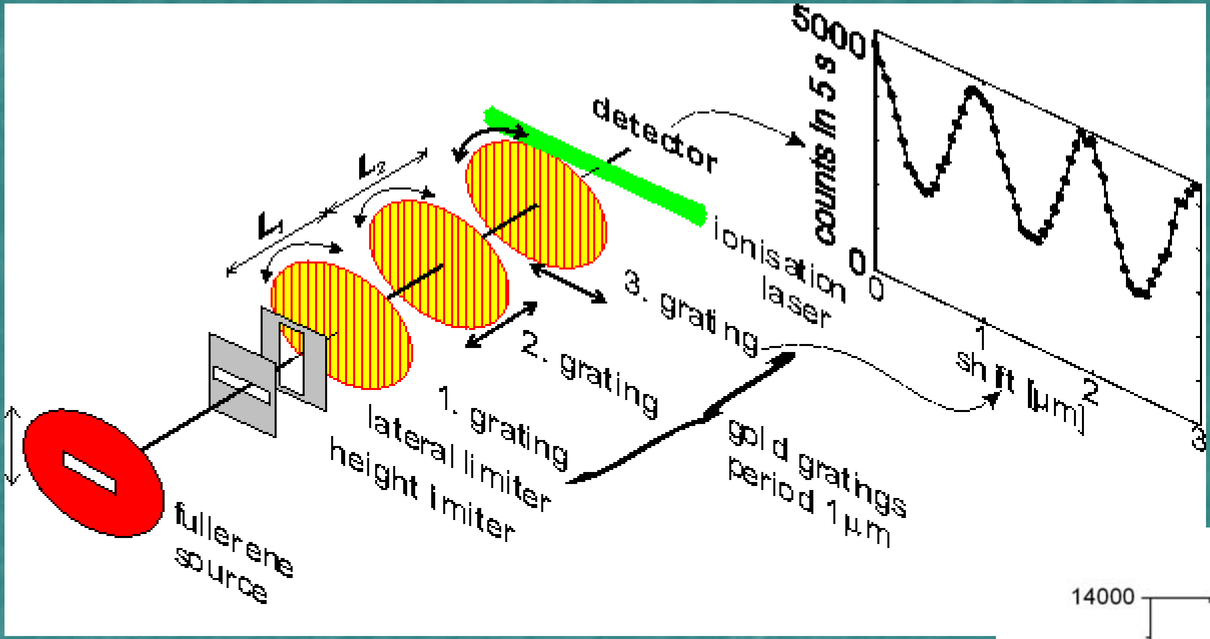
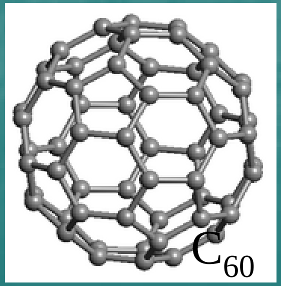


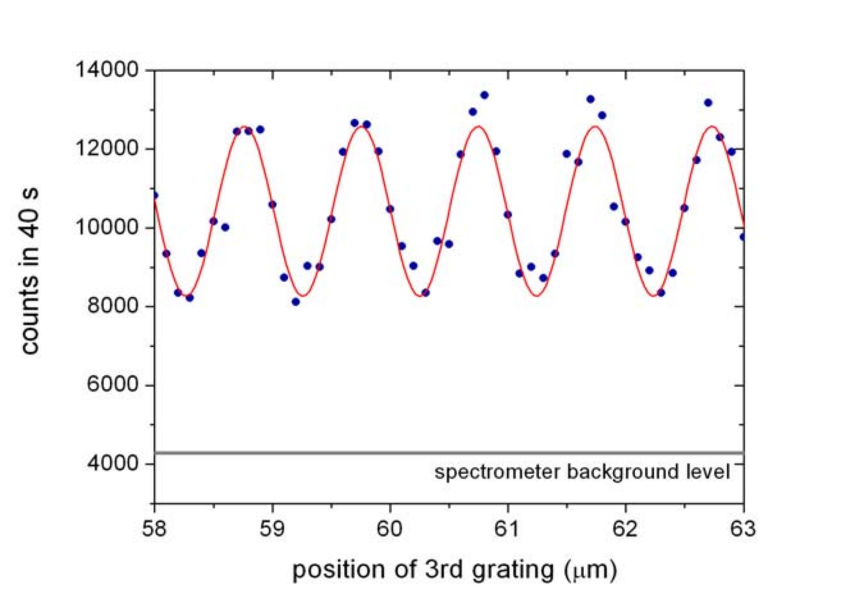


How Large Can a Particle Be in Order to Interfere?



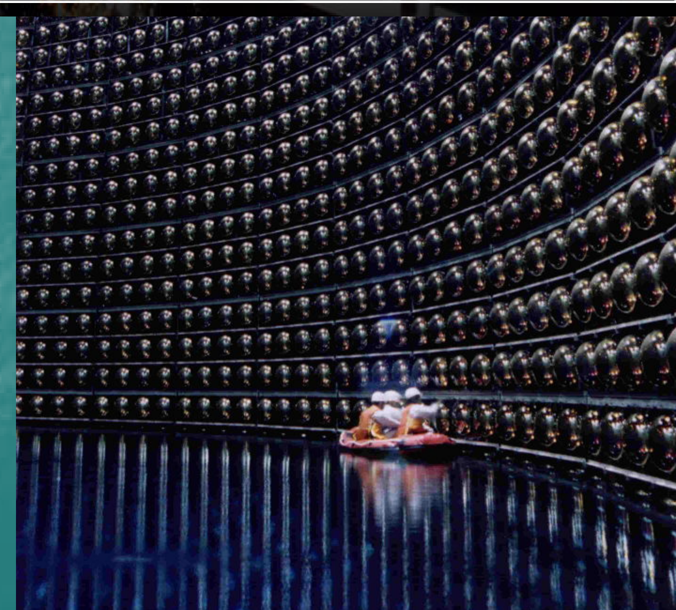
A.Zeilinger
Vienna University of Technology ‡





Same Observation for Light

Light is more likely to be regarded as a wave but it has particle properties as well.
⇒ photoelectric effect



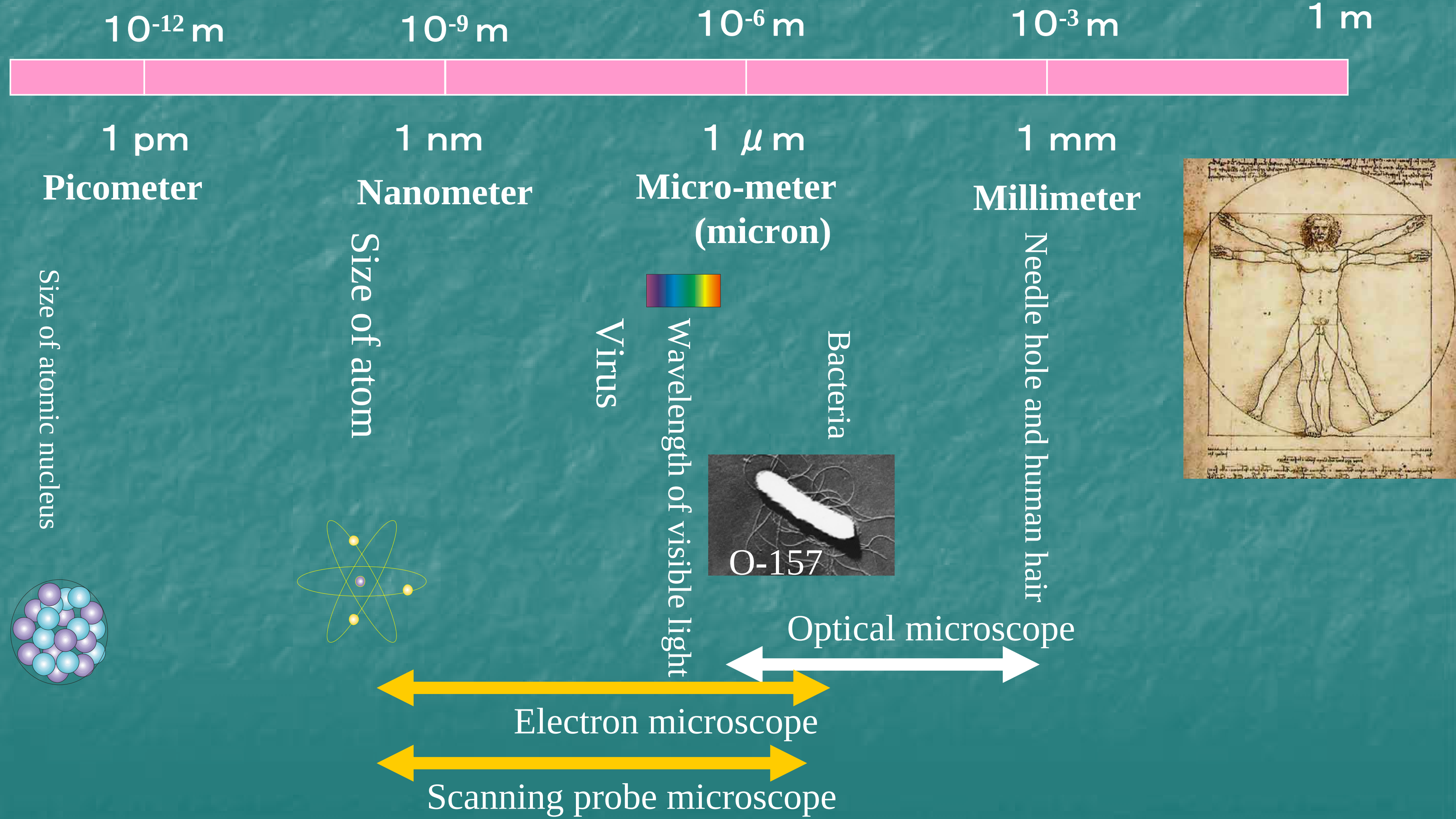
Photomultiplier tube catches faint light.

A single photon can be detected.

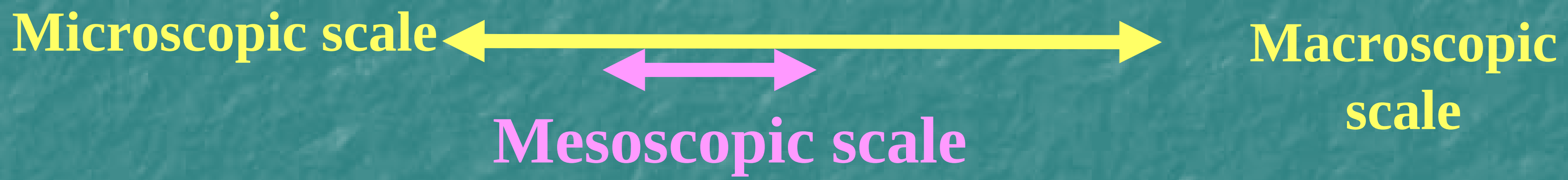
- Topics we have covered up to now only deal with quantum interference in a vacuum state.
- Do electrons in matter perform quantum interference too?
- Quantum interference properties: coherence.
- Decoherence makes for a loss of coherence in the system.

Artificial Materials and Mesoscopic Systems

The Microscopic Universe



The Mesoscopic System



It is an intermediate scale between microscopic and macroscopic systems.

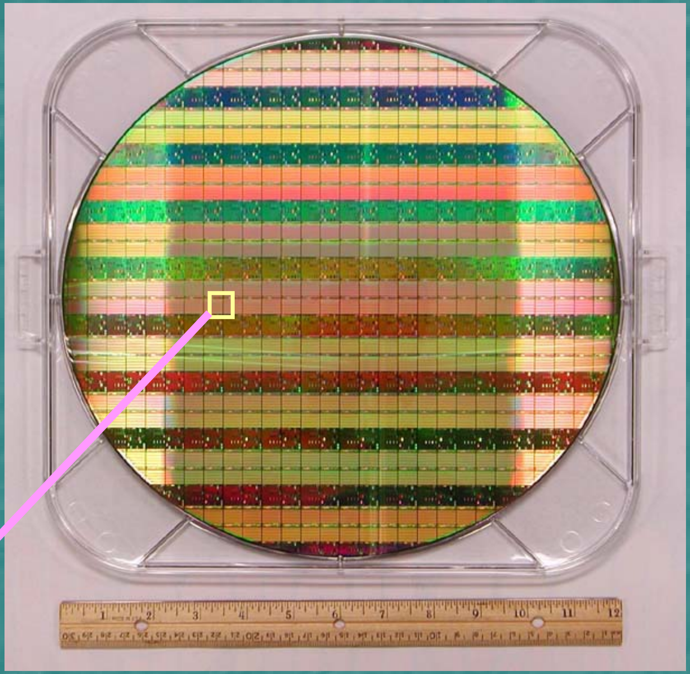
Micro-processing for Semiconductor Devices

Picture removed due to copyright restrictions

The first transistor invented by Bell Labs in the U.S.A. (1946)

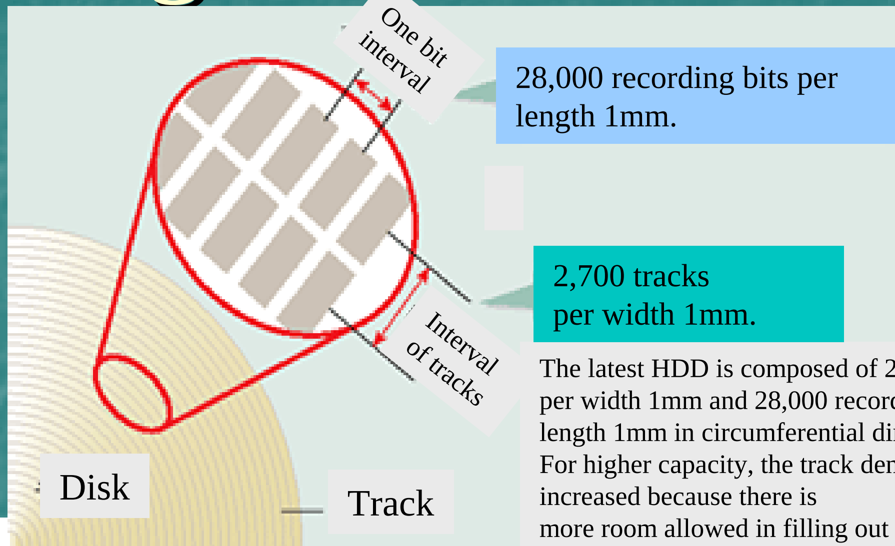
50 years

Silicon wafer



12 inch (30 cm)

Magnetic Recording Media

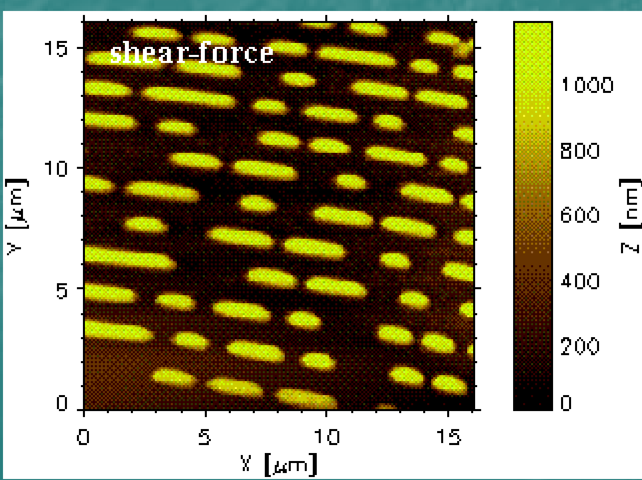


The latest HDD is composed of 2,700 tracks per width 1mm and 28,000 recording bits per length 1mm in circumferential direction.
For higher capacity, the track density is increased because there is more room allowed in filling out the tracks.

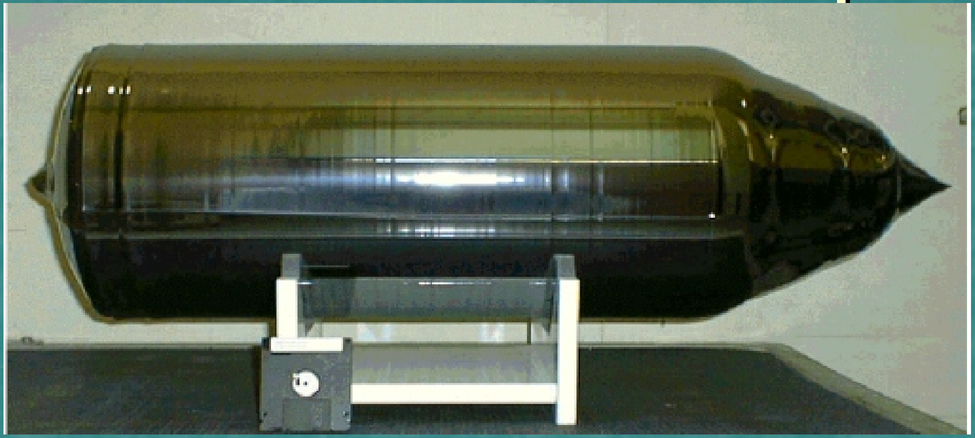
Hard disc

MO disc

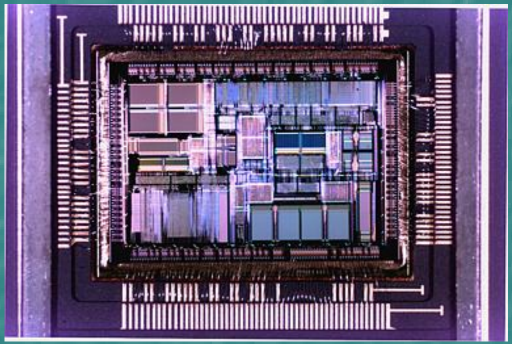
Information recording method that takes advantage of magnetization directions in a small magnetic body.
Recording unit (bit) of a high capacity HD is: 0.1mm×1mm



Silicon Monocrystal ⇒ Wafer ⇒ Super LSI

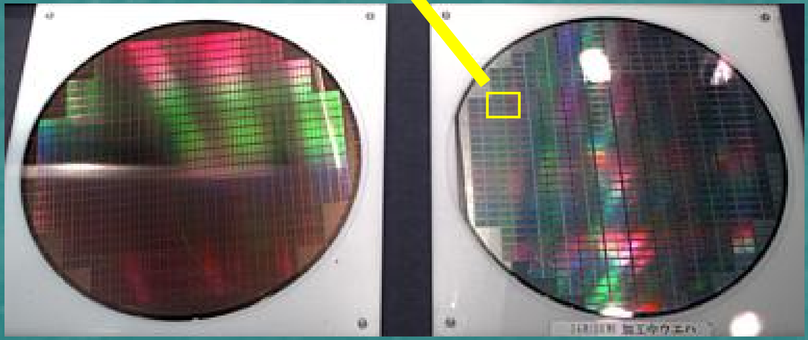


Super LSI
(Large Scale Integration)



Cutting out of the monocrystal.

Wafer



Micro-processing

Ultra-large Scale Integration Semiconductor (ULSI)

Moore's law

A prediction of Gordon Moore that the number of transistors (degree of integration) on a semiconductor chip doubles every one and half to two years.

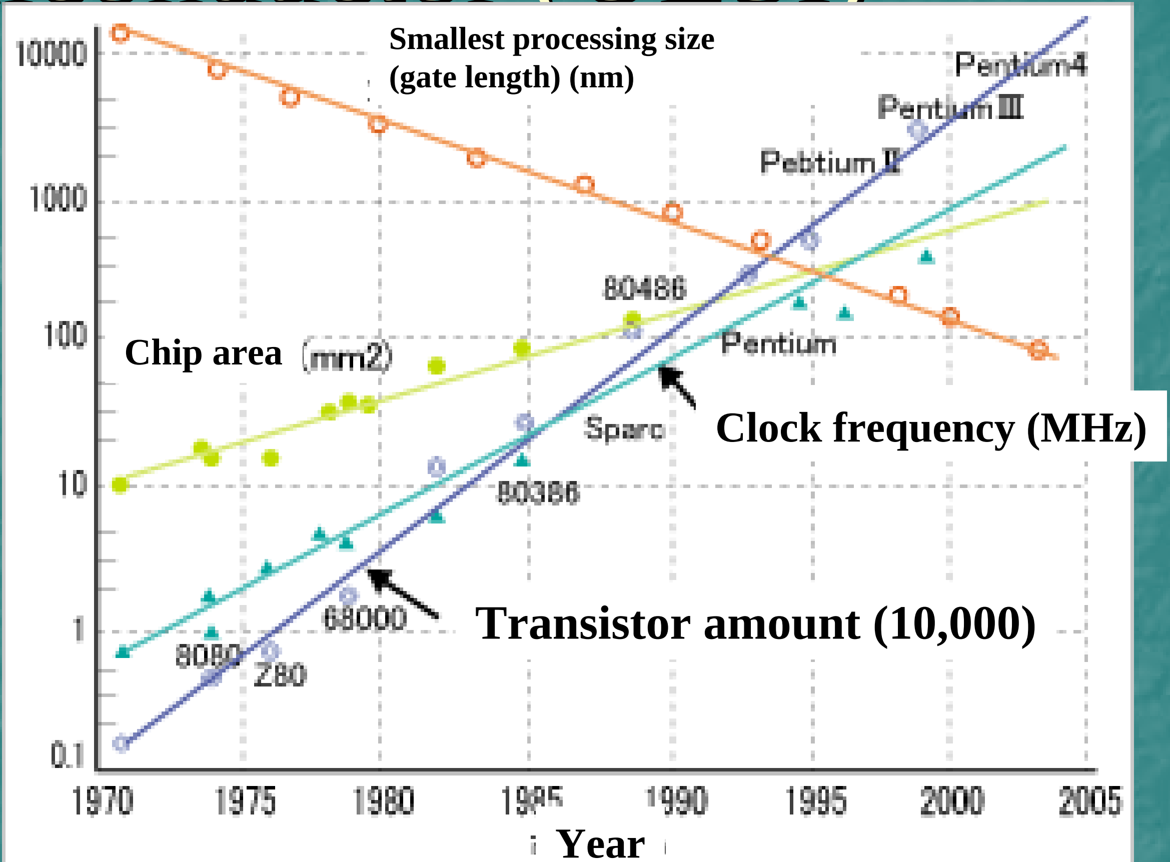


Figure 1. Development of LSI (Microprocessor)

It is clear that there is a limit to the micro-processing technique because semiconductor devices have already reached submicron scale (less than 1 μm) .

The Mesoscopic System

Size of the system L

and

The lengths that are characteristic of physical phenomena

de Broglie wavelength λ

Free mean path ℓ

Phase relaxation length L_ϕ

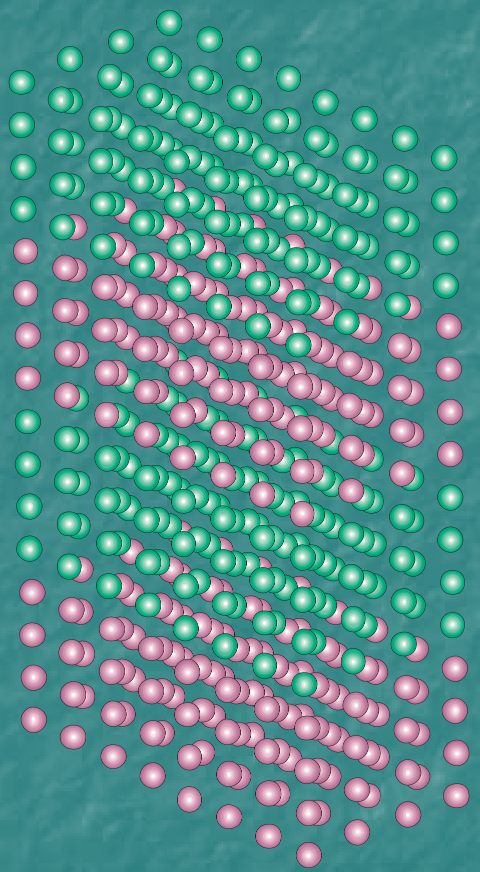
Region of submicron-scale

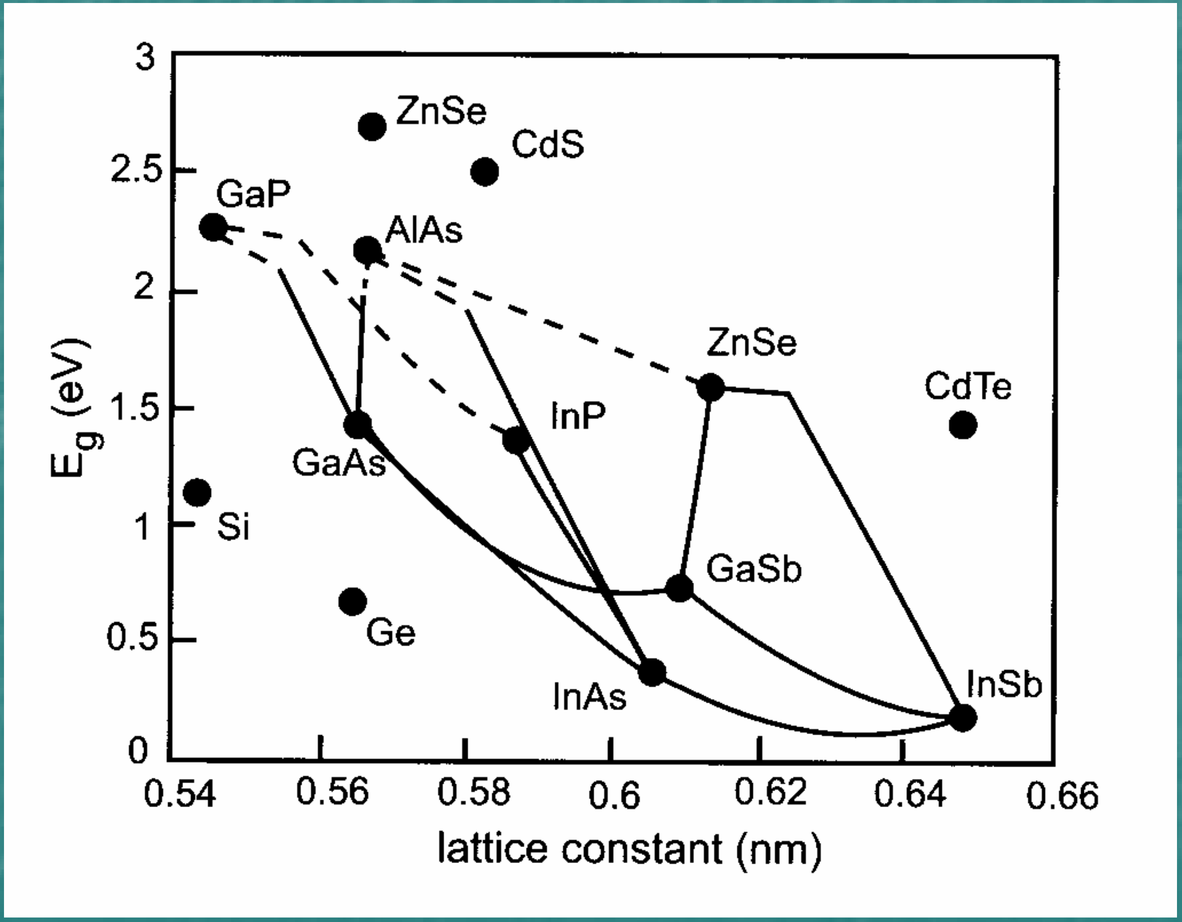
The behaviors are different from those of a microscopic system;
strong effects of quantum mechanics can be observed.

Conditions Necessary for Mesoscopic Physics and Nanoscience

- Artificial materials
- Micro-processing
- Extreme environments (extreme low temperatures and strong magnetic fields)
- High-sensitivity, high-resolution, and high-accuracy measurements

Artificial Superlattice



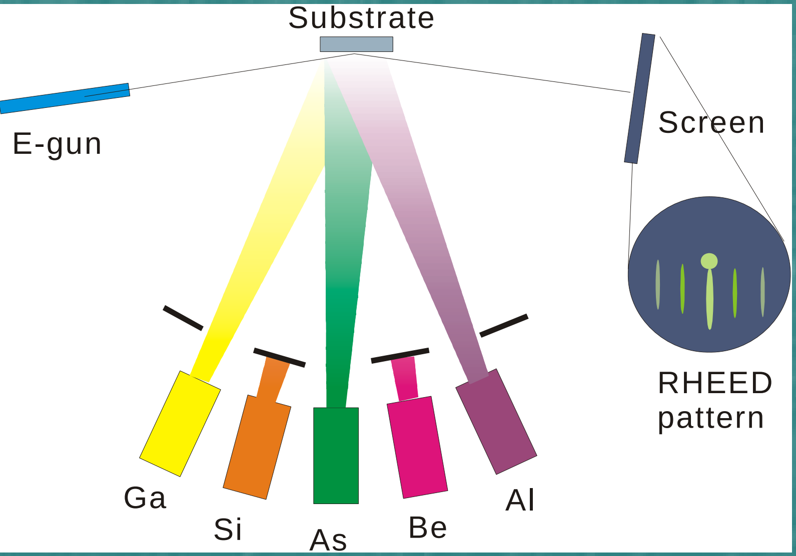


Dr. Leona Esaki (1972) ‡

Semiconductor Artificial Superlattice Production

Molecular beam epitaxy

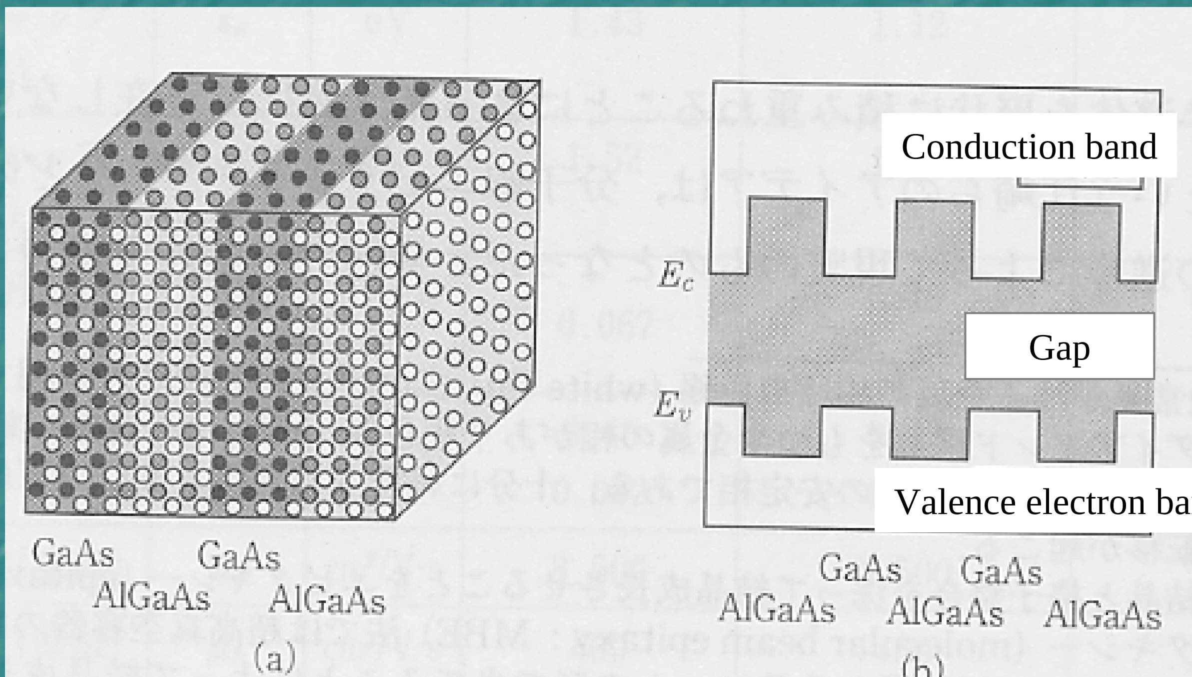
Accumulate each layer of atoms inside ultra-high-vacuum in a clean environment.



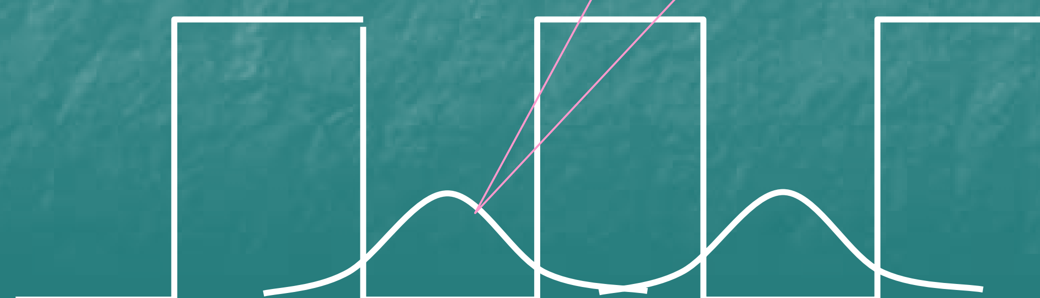
Iye Laboratory, The Institute for Solid State Physics, University of Tokyo. ‡

The artificial superlattice can be obtained by accumulating different types of atomic layers.

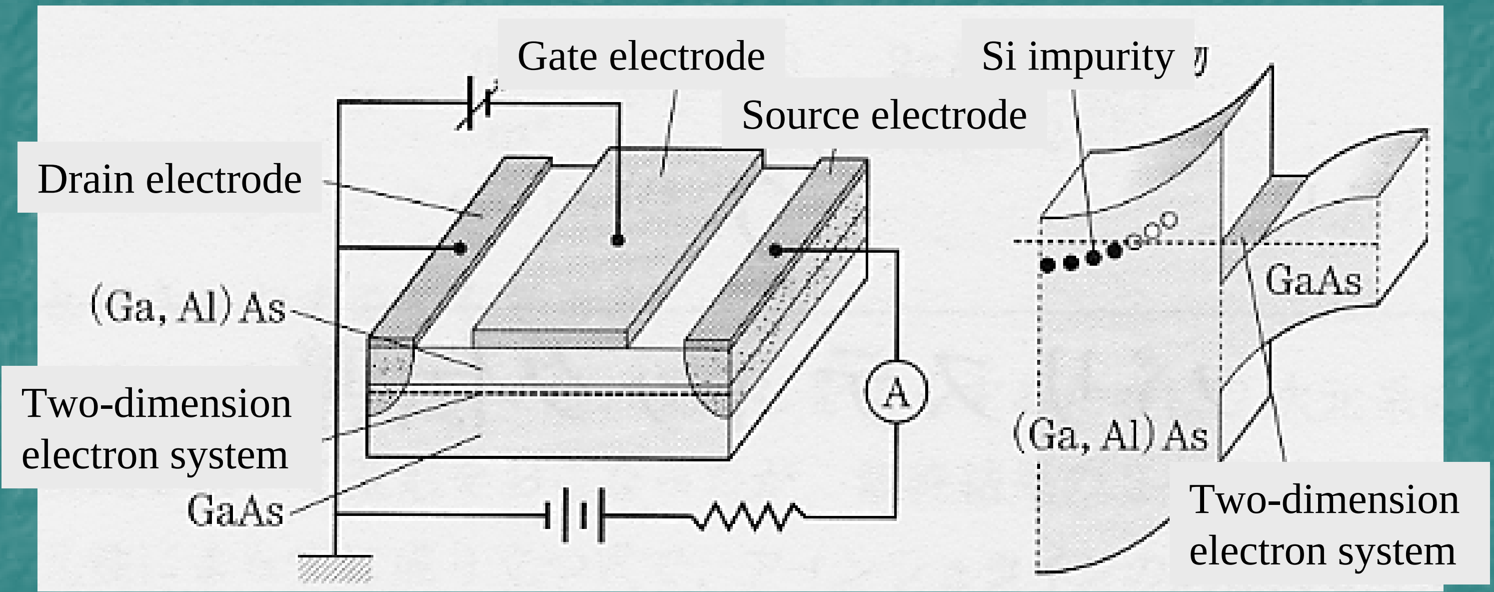
Potential of Artificial Superlattices



GaAs/AlGaAs



Two-dimension Electron System at Semiconductor Interface



The electron is confined by potential in the direction perpendicular to the interface while it is freely traveling in the direction parallel to the interface.

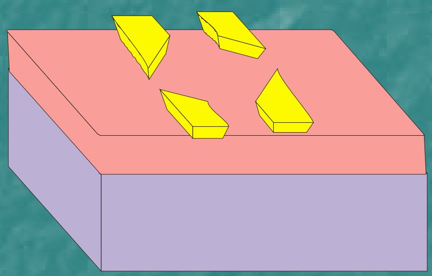
The electron density can be changed by applying electric voltage to the gate electrode.

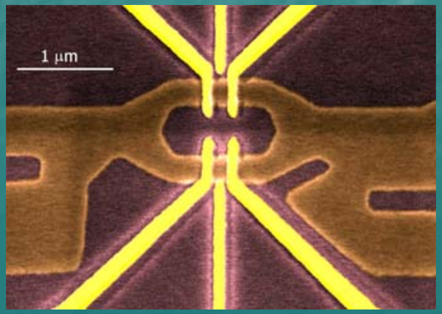
The potential for the electron is artificially induced by using an appropriate form of gate electrode.

Examples of Mesoscopic Samples

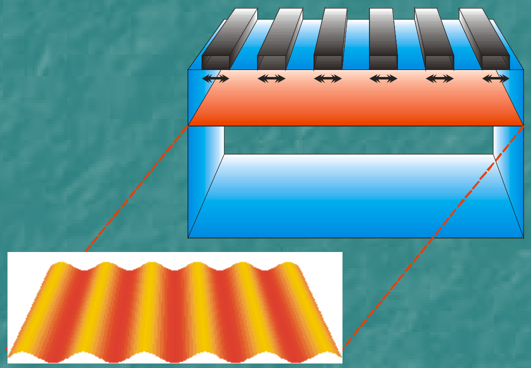
Variety of artificial structures are produced by micro-processing of two-dimensional electron system samples.

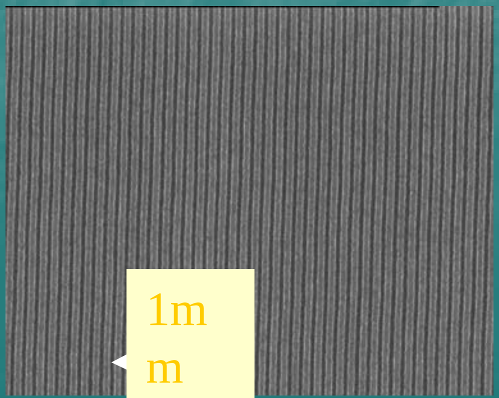
Quantum point contact
Quantum dot



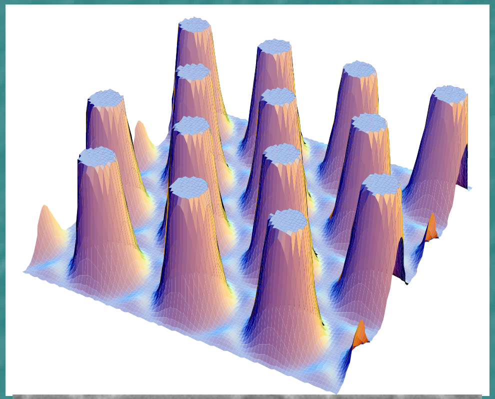


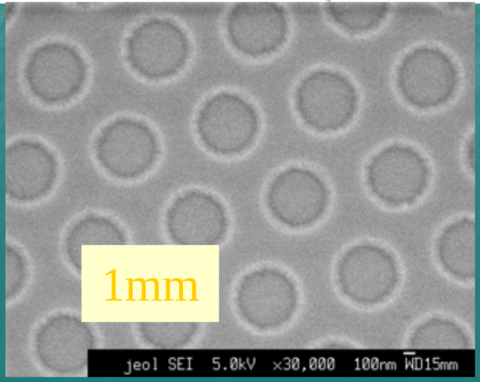
Single-dimension modulation
(washboard potential)





Two-dimension modulat
(antidot lattice)

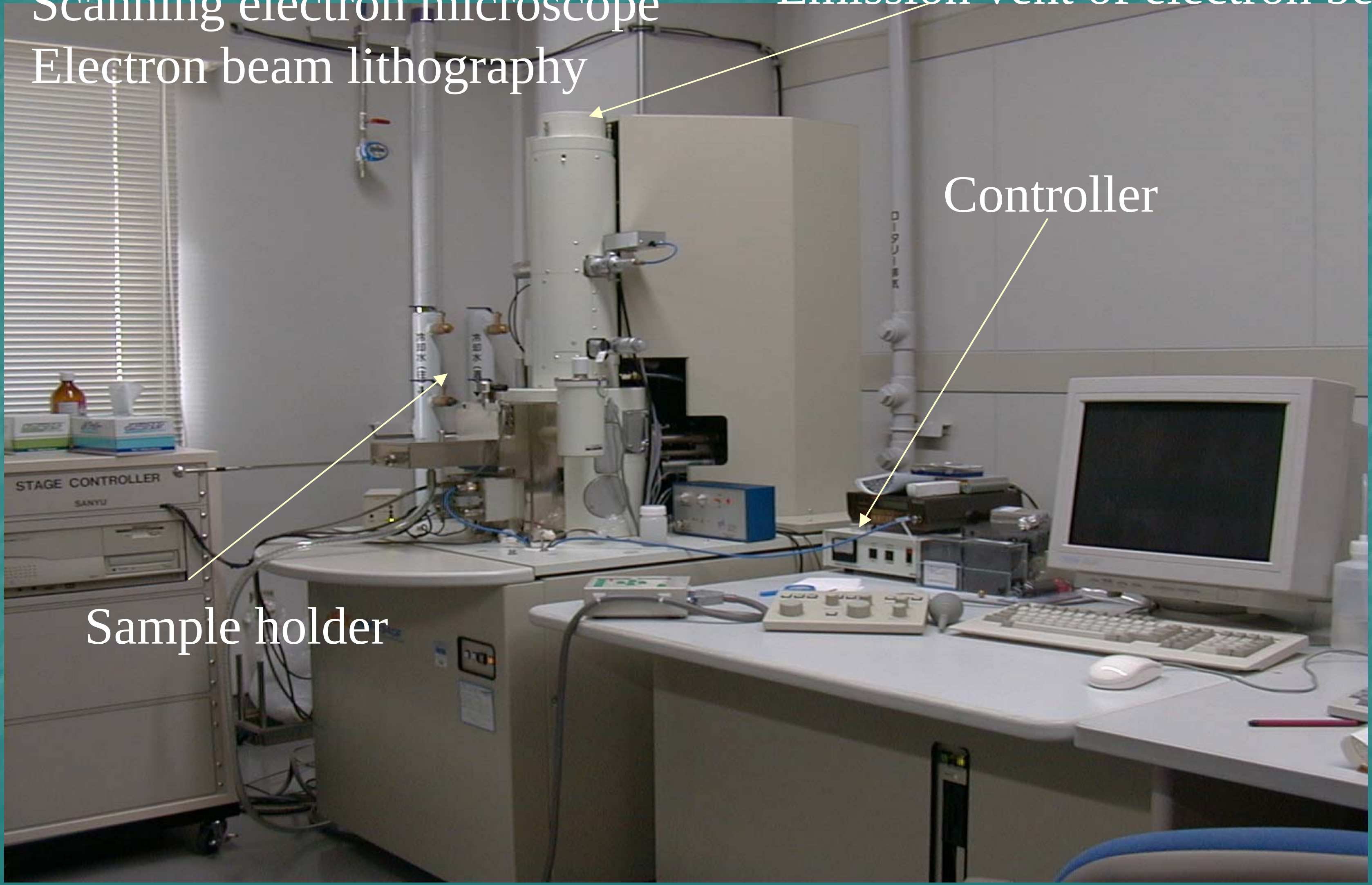




Clean Room Operation

Iye Laboratory, The Institute for Solid State Physics, University of Tokyo. ‡

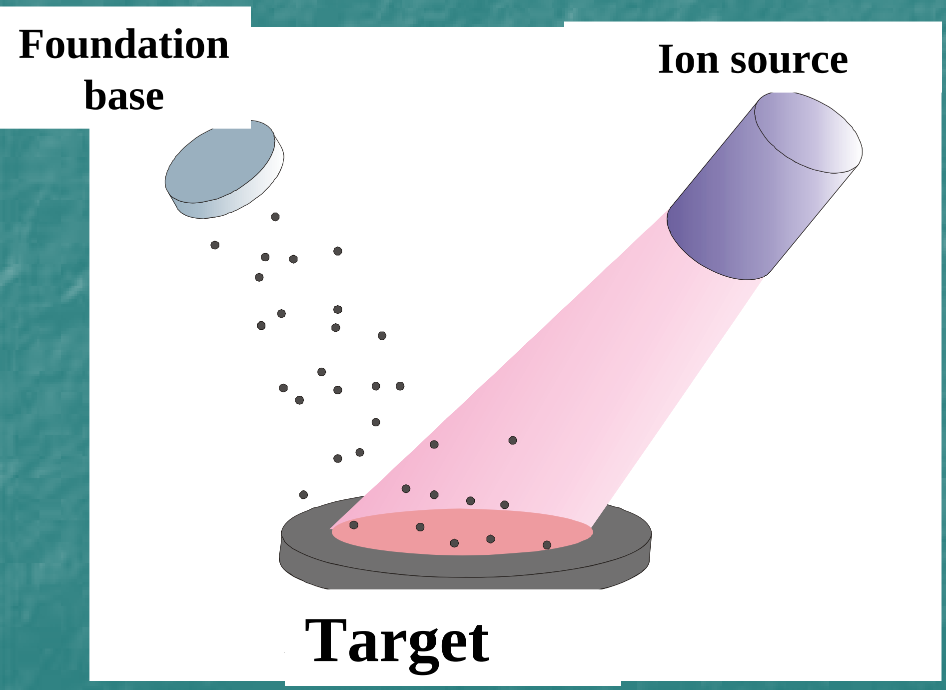
Electron Beam Lithography System



Iye Laboratory, The Institute for Solid State Physics, University of Tokyo.

Metal Vapor Deposition

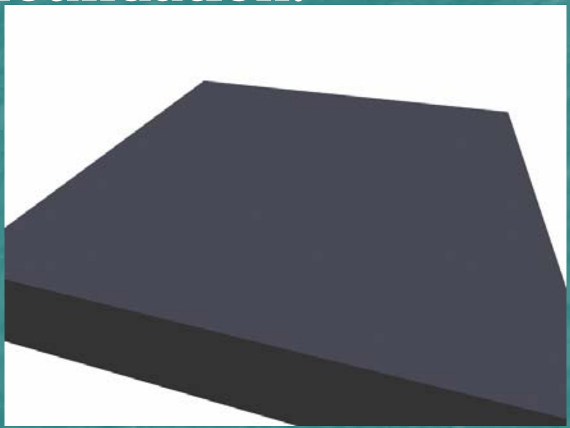
Ion beam spattering system



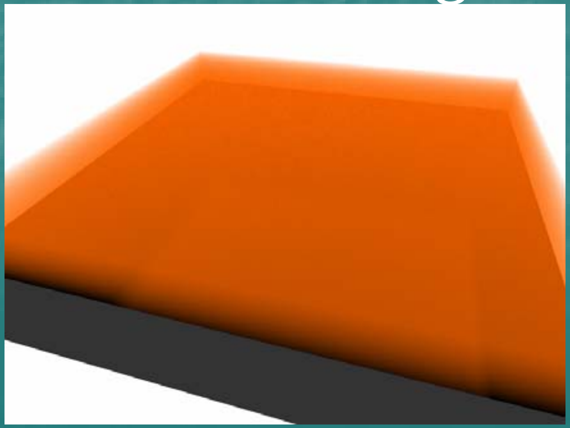
Iye Laboratory, The Institute for Solid State Physics, University of Tokyo. ‡

Formation of Micro-patterns

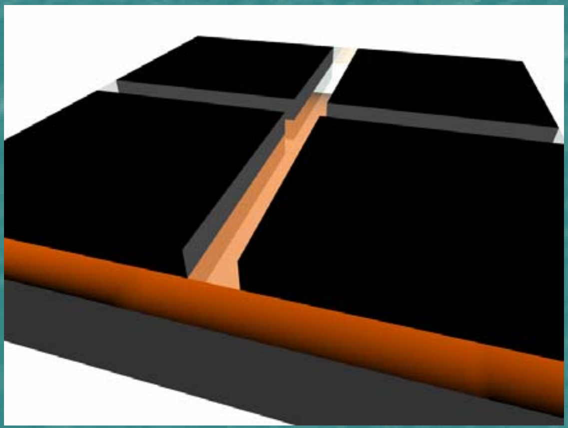
① Cleansing the foundation.



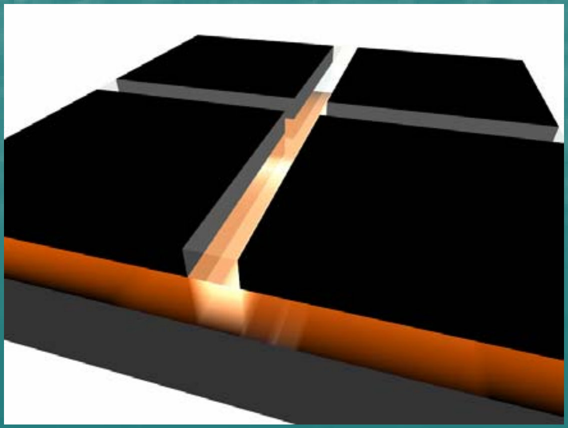
② Resist coating



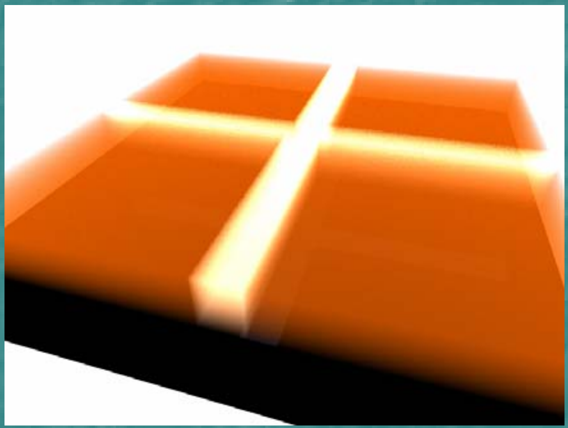
③ Mask alignment



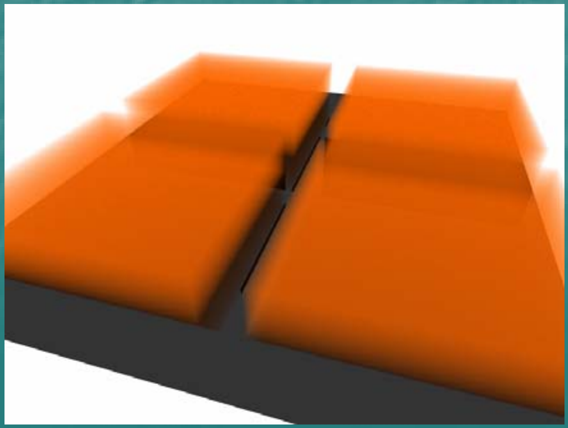
④ Exposure



⑤ After exposure

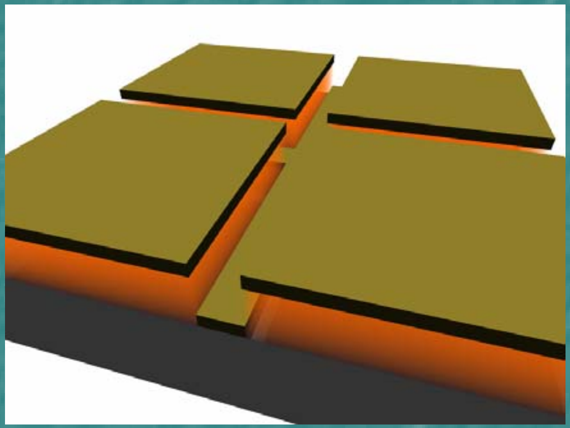


⑥ Development

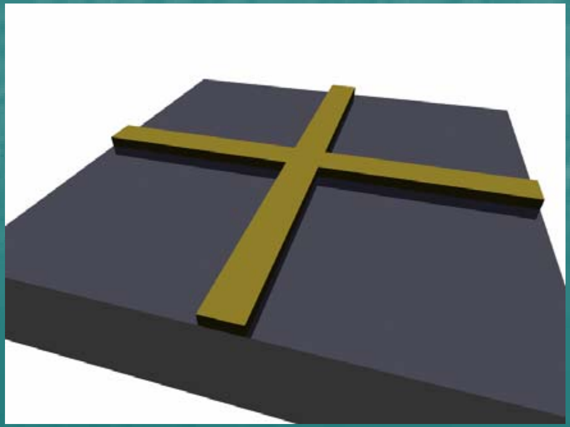


Vapor Deposition and Lift-off

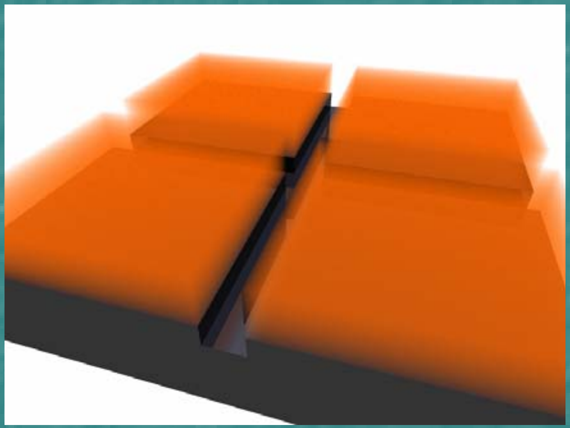
Vapor deposition



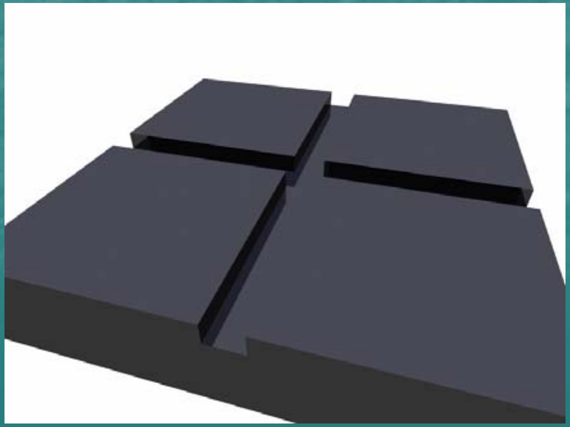
Lift-off



Etching



Resist removal

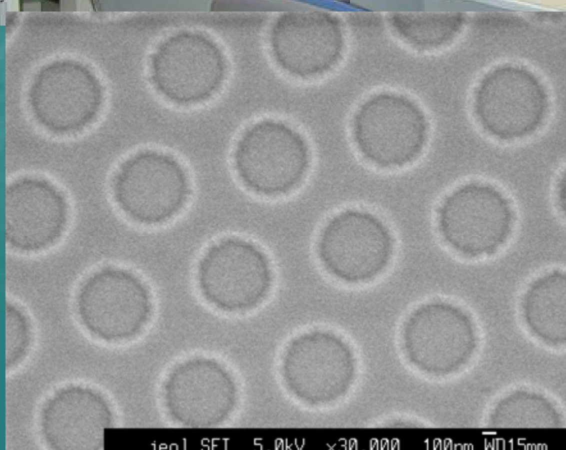


Production of Quantum Structures by Ultra-micro-processing

Electron lithography system

The quantum dot was produced by micro-processing of the electron lithography.

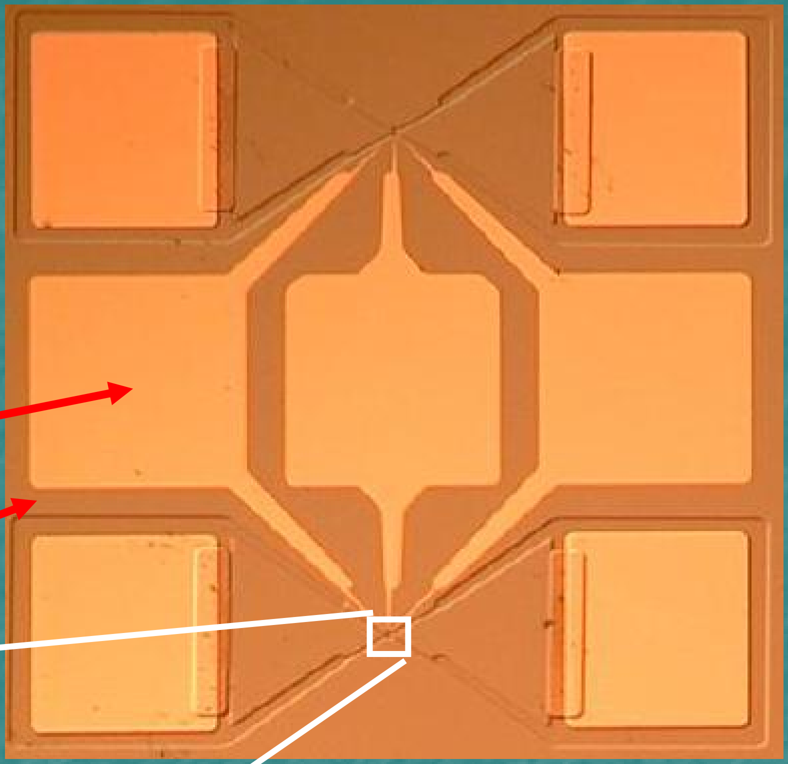
Iye Laboratory, ISSP, University of Tokyo.



The antidot lattice of two-dimension electron system in a semiconductor: Artificial superperiodic structure.

Micro-structure Sample

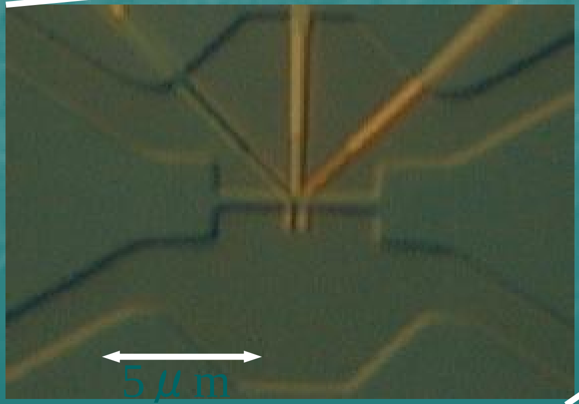
The example for the micro-structure sample observed by an optical microscope. A single quantum dot is formed both above and below in the structure.



1.2 mm

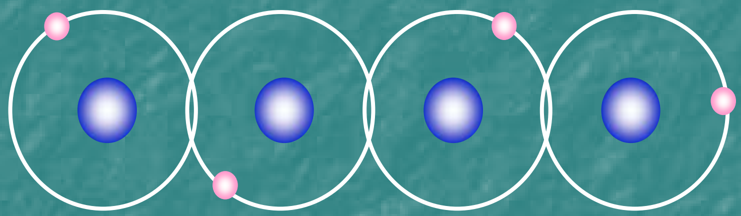
Gold deposited electrode

Etching-treated part

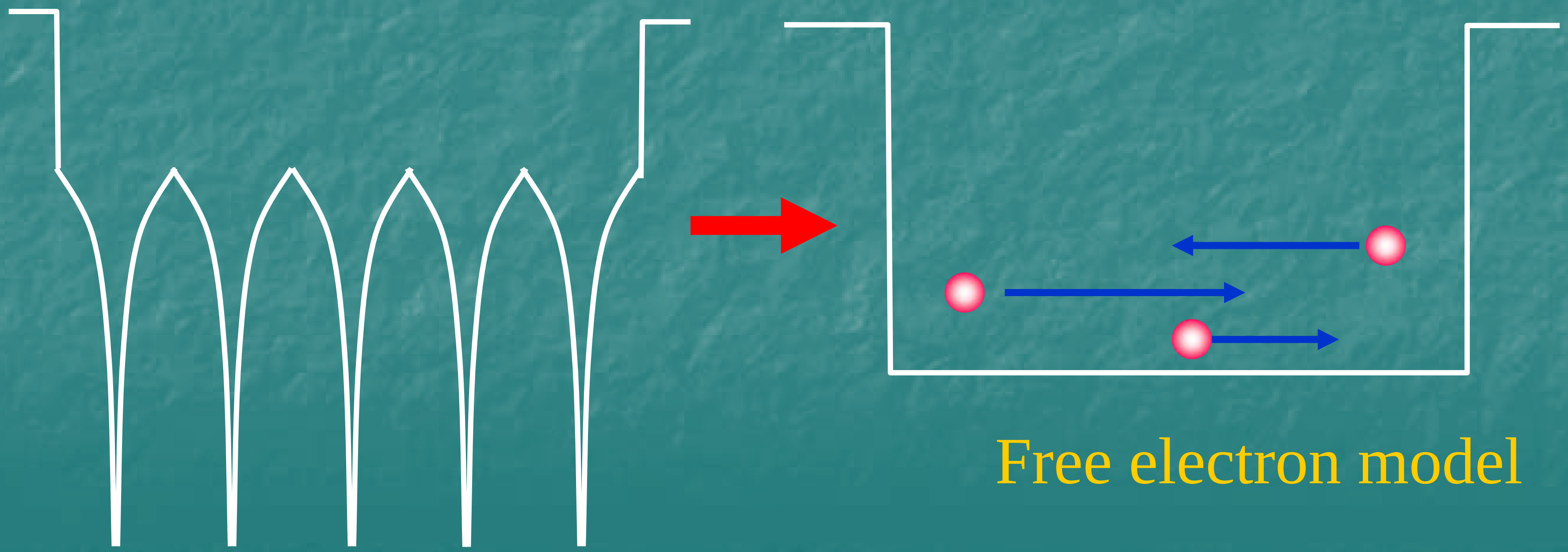


Quantum Conduction

Conduction Electrons in Metals

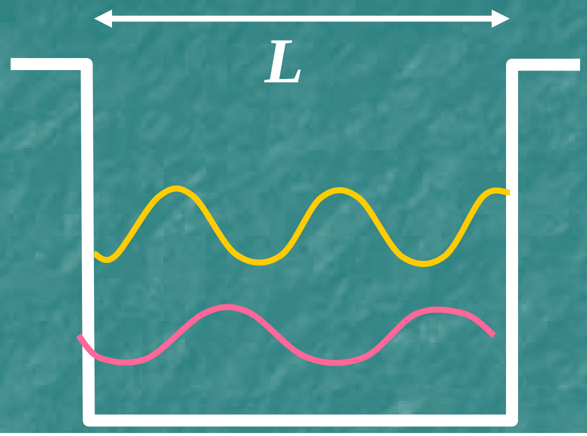


In metals, there is an electron (conduction electron) that can move around in the entire crystal.



Free electron model

Quantum Mechanics: Particles in a Box



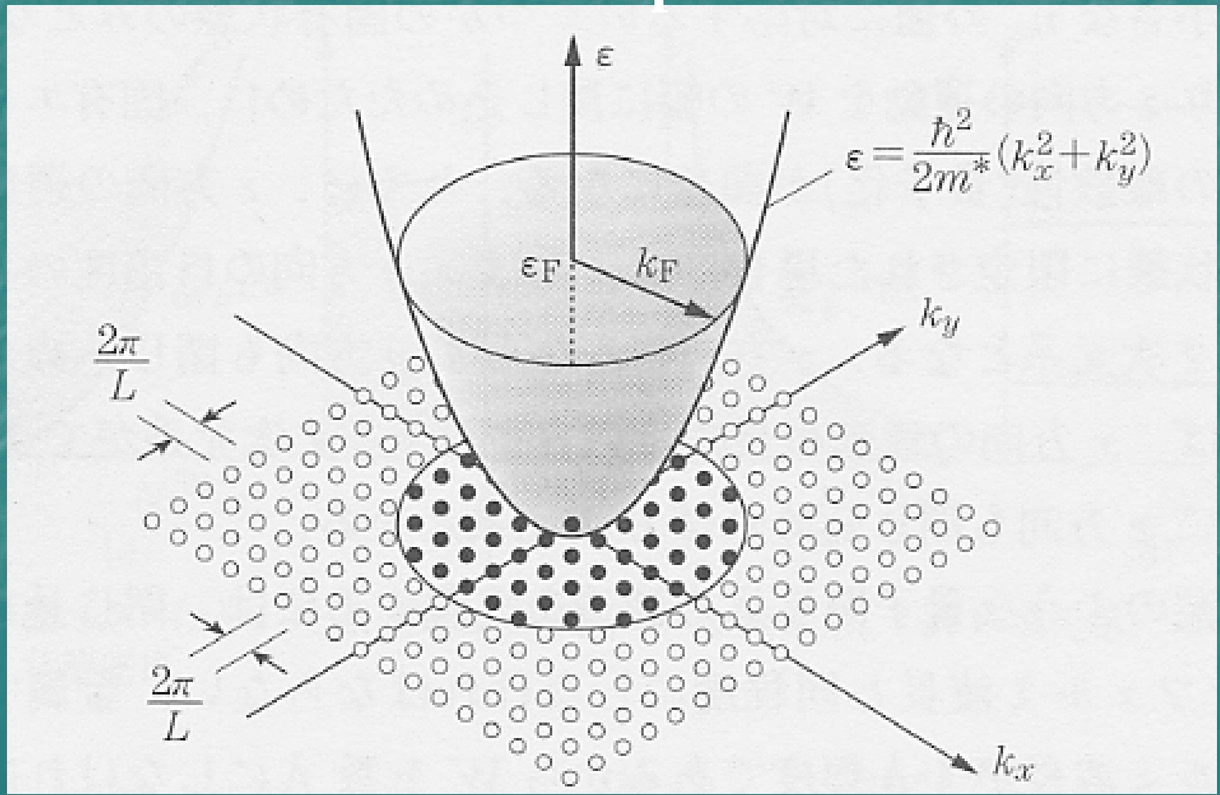
$$\psi(\mathbf{r}) = \frac{1}{\sqrt{L^3}} e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$\varepsilon = \frac{\hbar^2 \mathbf{k}^2}{2m}$$

Permissive state of wave number space.

$$k_x = \frac{2\pi}{L} n_x \quad (n_x = 0, \pm 1, \pm 2, \cdots)$$

The permitted wave number of an electron.



Electron is filled from lower energy level

⇒Fermi level

Fermi surface ε_F

and sphere

Electric Conduction

Electric current $\mathbf{J} = ne\langle \mathbf{v} \rangle$

Semi-classical equation of motion

$$m\frac{d\langle \mathbf{v} \rangle}{dt} = e\mathbf{E} - m\frac{\langle \mathbf{v} \rangle}{\tau}$$

Acceleration due to electric field.

"Friction" term due to scattering.

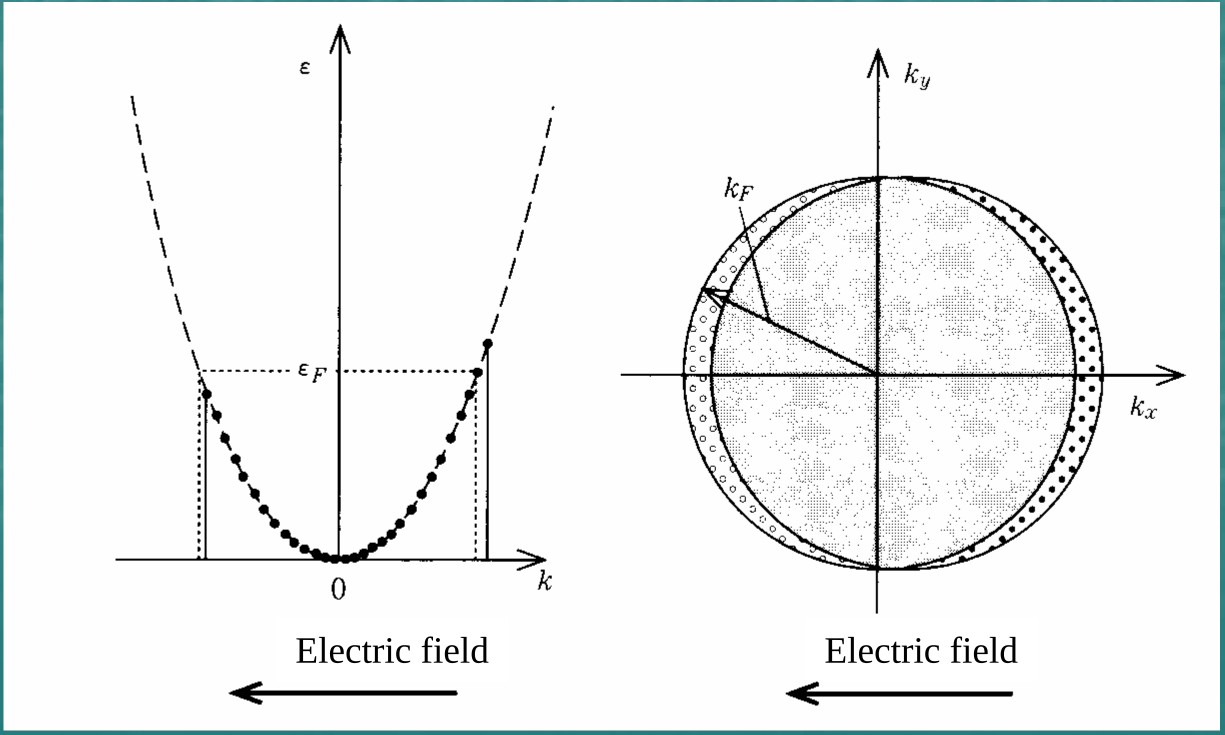
$$\langle \mathbf{v} \rangle = \frac{e\tau}{m}\mathbf{E} = \mu\mathbf{E}$$

Electron mobility

$$\mathbf{J} = \frac{ne^2\tau}{m}\mathbf{E} = \sigma\mathbf{E}$$

$$\sigma = \frac{ne^2\tau}{m}$$

Drude formula



Velocity of Electrons

Make s current of one ampere into lead wire with cross section area 1 mm^2

$$\mathbf{J} = ne\langle \mathbf{v} \rangle = 10^6\ \text{A/m}^2$$

Conduction electron density of metal:

$$n = 10^{29}\ \text{m}^{-3}$$

$$\langle v \rangle = \frac{J}{ne} = \frac{10^6\ \text{Am}^{-2}}{10^{29}\ \text{m}^{-3} \times 1.6 \times 10^{-19}\ \text{C}} \approx 6 \times 10^{-5}\ \text{m/s} = 0.06\ \text{mm/s}$$

Speed of snails??

This is the average speed.

$$\varepsilon = \frac{\hbar^2 \mathbf{k}^2}{2m}, \quad \mathbf{v}_{\mathbf{k}} = \frac{1}{\hbar}\frac{\partial \varepsilon}{\partial \mathbf{k}}$$

The velocity of an electron at the Fermi level

$$v_F \approx 10^6\ \text{m/s}$$

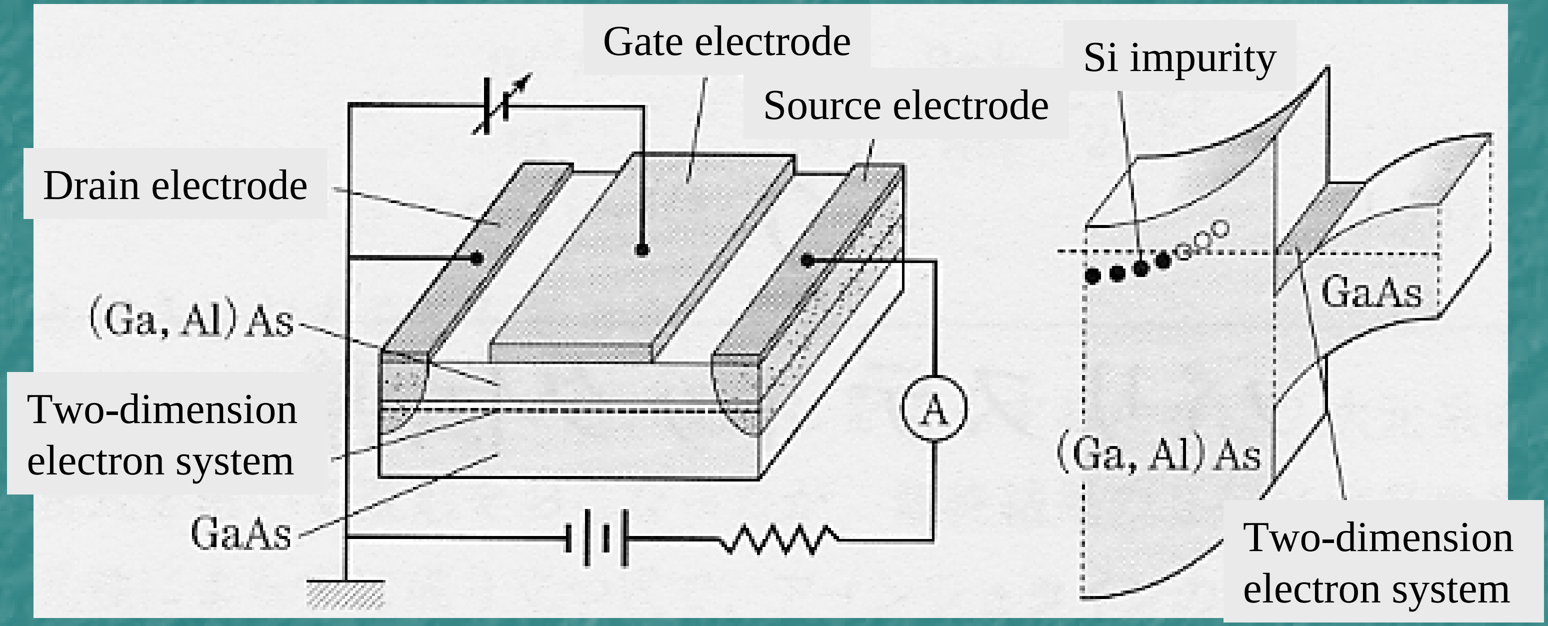
For ordinary metals:

(one hundredth the speed of light)

If there is no electric field, electrons travel in both directions positive and negative to cancel each other out, and as a result, the net electric current is zero.

Even a subtle loss of balance can create very large electric current.

Two-dimension Electron System at Semiconductor Interface



Electron density $n = 10^{13} \sim 10^{16}\,\mathrm{m}^{-2}$

HEMT (High Electron Mobility Transistor)

High Mobility

Electron mobility

$$\mu > 1000\ \mathrm{m}^2/\mathrm{Vs}$$

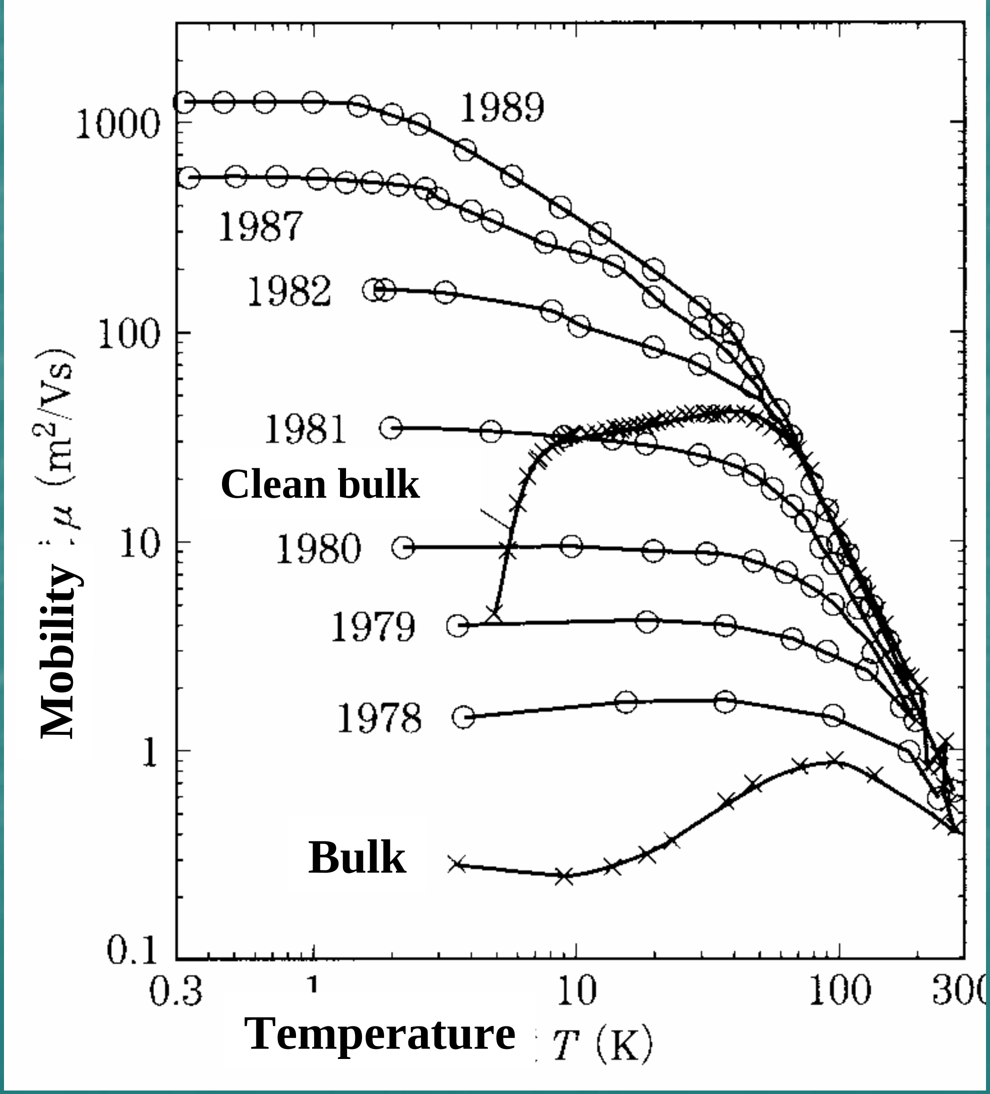
Mean free path

$$\ell > 100\ \mu\mathrm{m}$$

Micro-processing lets us to make samples that are smaller than those shown in the figure.

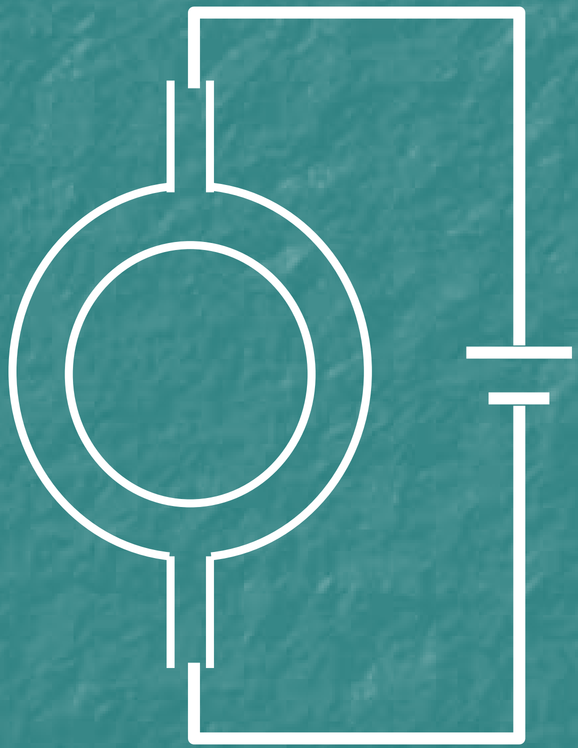
Ballistic conductivity

$\ell > L$ （L: sample size）



Mesoscopic Circuit

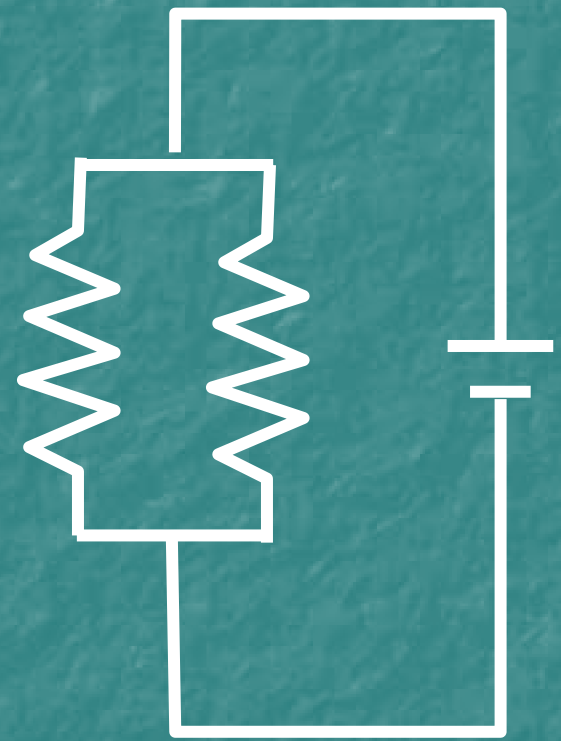
Mesoscopic ring



$$G \neq G_R + G_L$$

$\neq$

Classical parallel circuit



$$G = G_R + G_L$$

Quantum Resistance

The quantity of the dimension where the electric resista
belongs is made of physical constants.

e Electric charge e ＝electric current × time

$\frac{h}{e}$ Magnetic flux Φ ＝voltage × time

$\frac{h}{e^2}$ Magnetic flux/electric charge
＝ voltage/electric current ＝ electric resistance

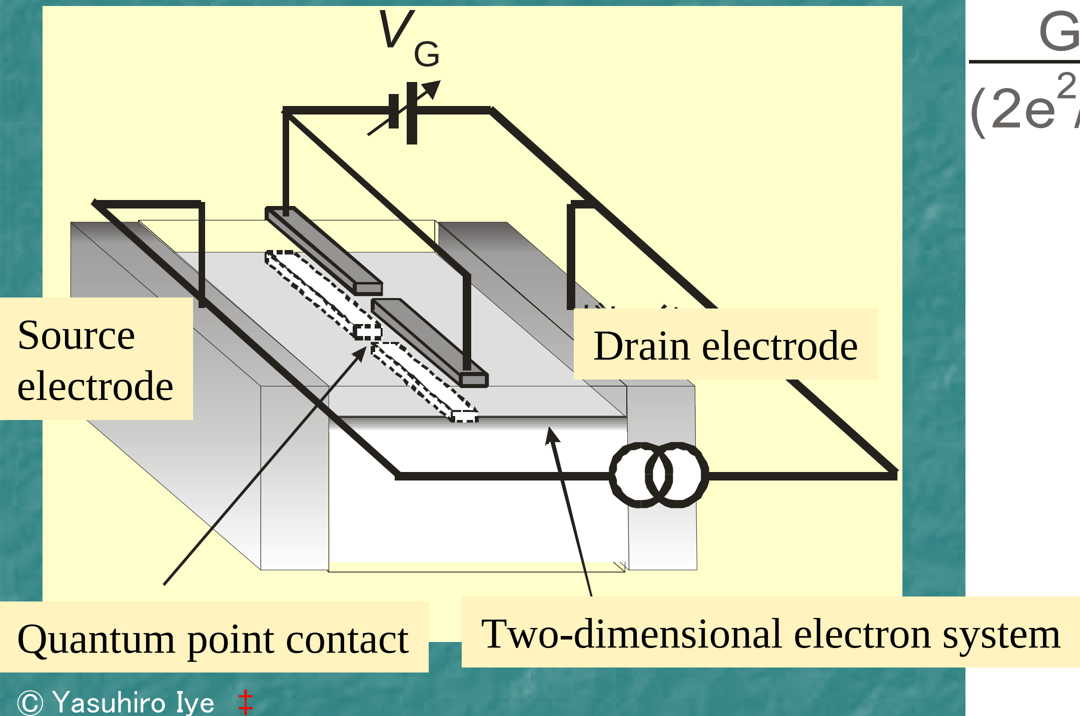
$$\frac{h}{e^2} = 25.813\ \mathrm{k}\Omega$$

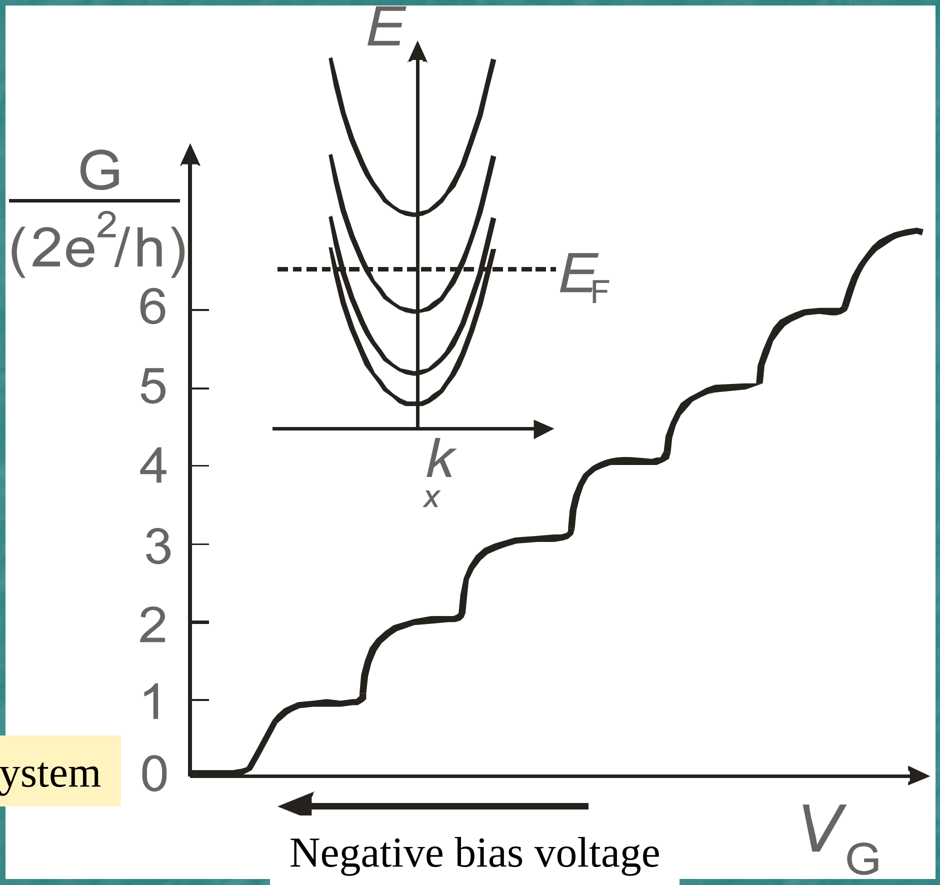
$$\frac{e^2}{h} = \frac{1}{25{,}813\ \mathrm{k}\Omega} = 38.7\ \mu\mathrm{S}$$

Quantum resistance   Quantum conductance

Conductance Quantization

Quantum point contact



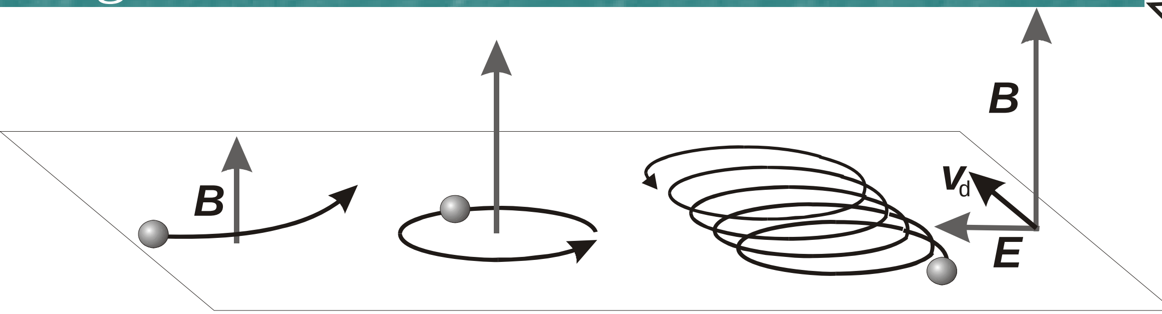


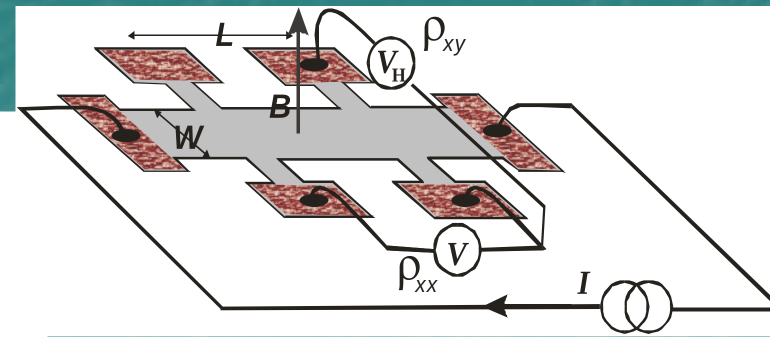
$$\frac{e^2}{h} = \frac{1}{25.813\,\mathrm{k}\Omega}$$

$$G = \frac{1}{R} = N\frac{2e^2}{h}$$

Hall Effect

Behavior of electrons in an electrostatic magnetic field.





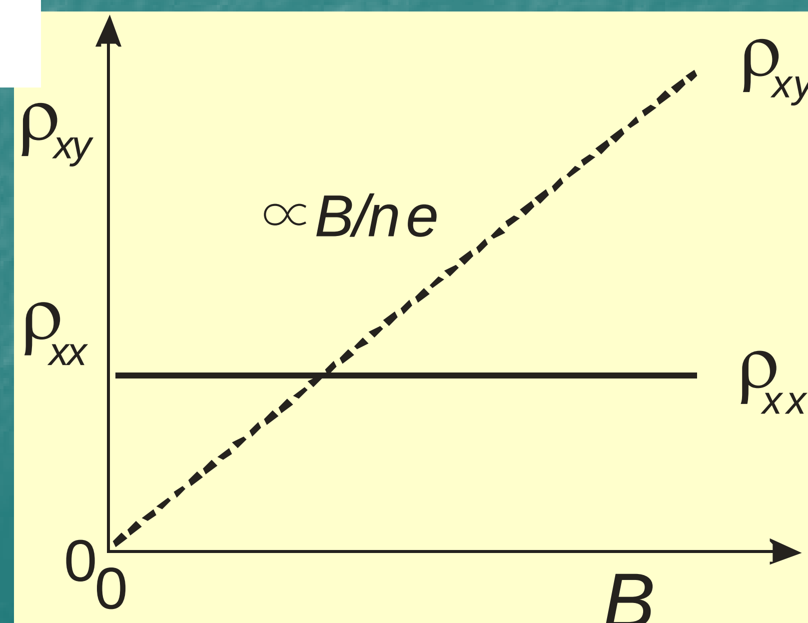
$$\hbar \frac{d\,\mathbf{k}}{dt} = e\left(\mathbf{E} + \mathbf{v}_{\mathbf{k}} \times \mathbf{B}\right) \qquad \mathbf{v}_{\mathbf{k}} = \frac{\partial\, \varepsilon_{\mathbf{k}}}{\partial\, \mathbf{k}}$$

Lorentz force

$$\rho_{xy}(B) = \frac{B}{ne} \qquad \rho_{xx}(B) = \rho$$

$$\mathbf{J} = \sigma \mathbf{E} \quad \Rightarrow \quad \begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\mathbf{E} = \rho \mathbf{J} \quad \Rightarrow \quad \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix}$$



Quantum Hall Effect

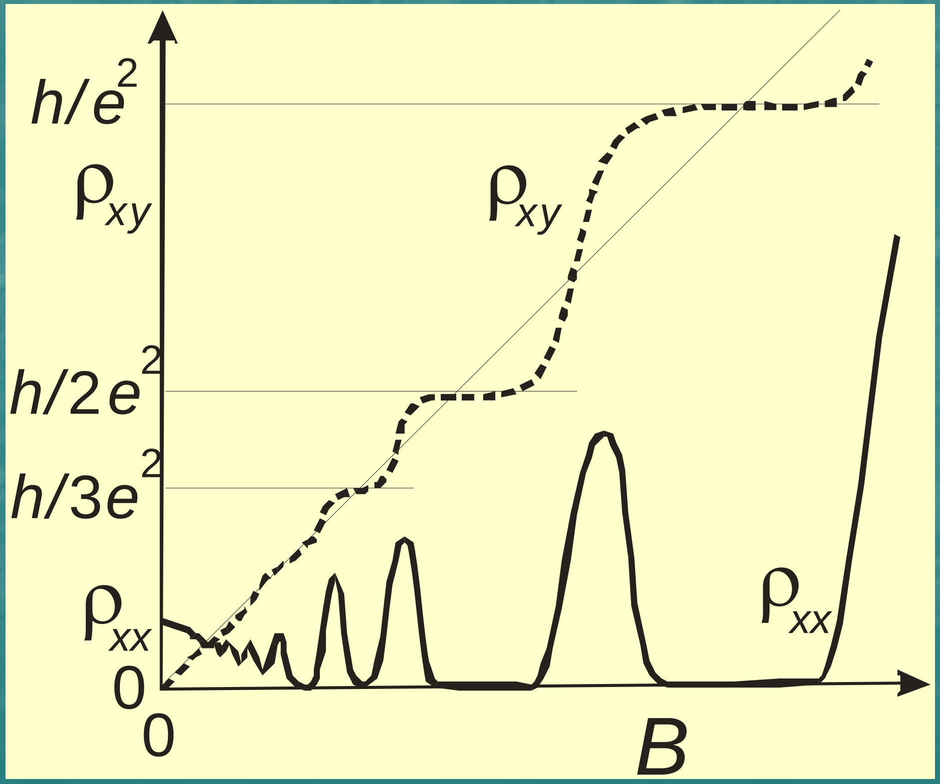
$$\rho_{xx} = 0$$

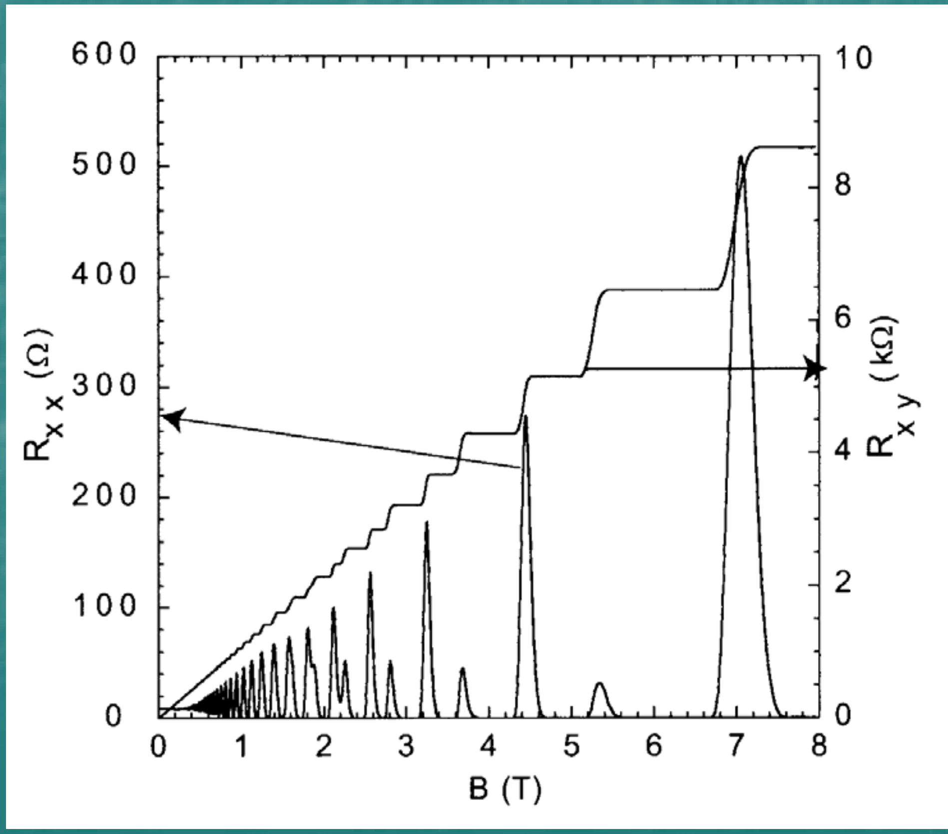
$$\rho_{xy} = \frac{1}{N}\frac{h}{e^2}$$

$$\frac{h}{e^2} = 25.813\,\mathrm{k}\Omega$$

With accuracy of more than eight digits, agrees with h/e^2.

$\Rightarrow$ International standard of resistance





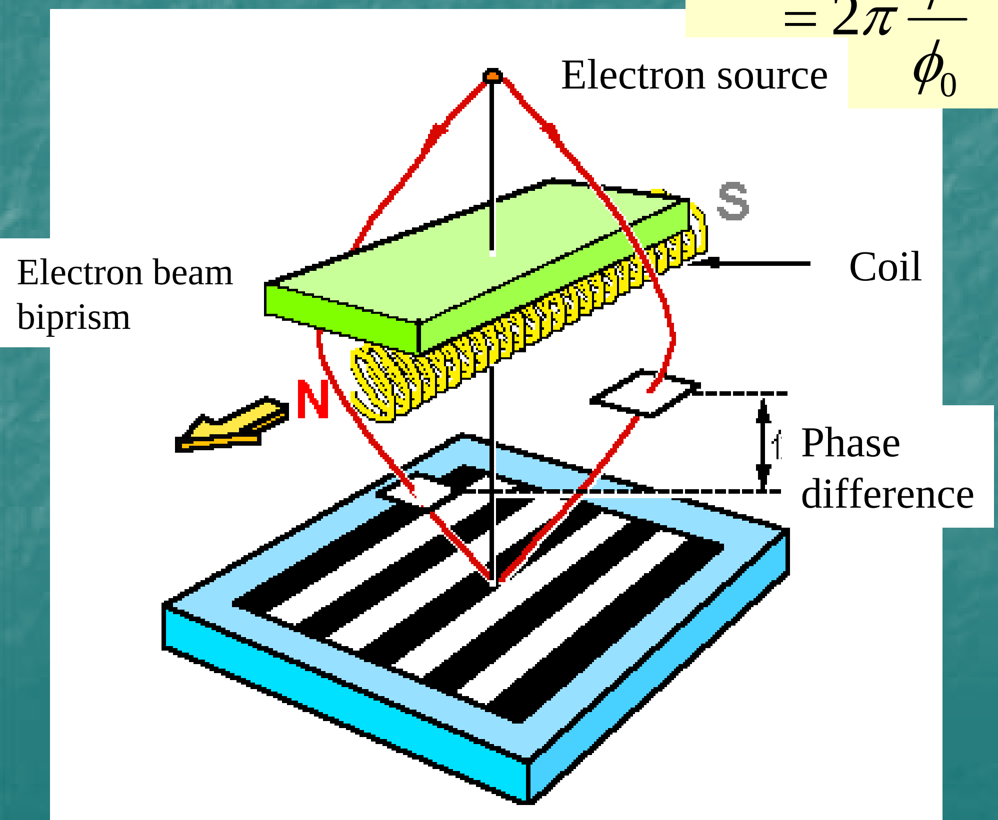
Aharonov-Bohm Effect (AB Effect)

Phase change in electron occurs due to the magnetic field (vector potential).

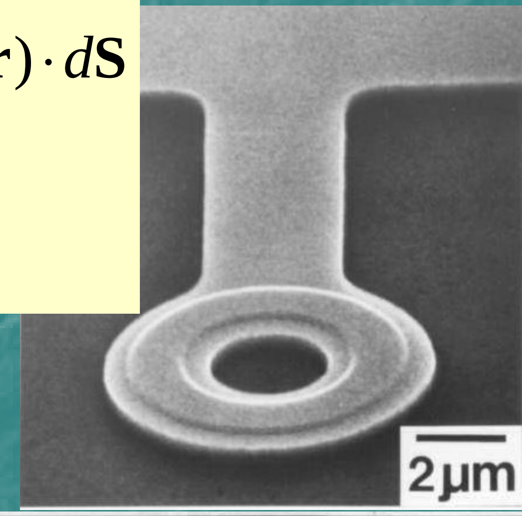
$$\Delta\theta = \frac{\hbar}{e}\int_{\mathrm{L}} \mathbf{A}(\mathbf{r})\cdot d\mathbf{r} - \frac{\hbar}{e}\int_{\mathrm{R}} \mathbf{A}(\mathbf{r})\cdot d\mathbf{r}$$

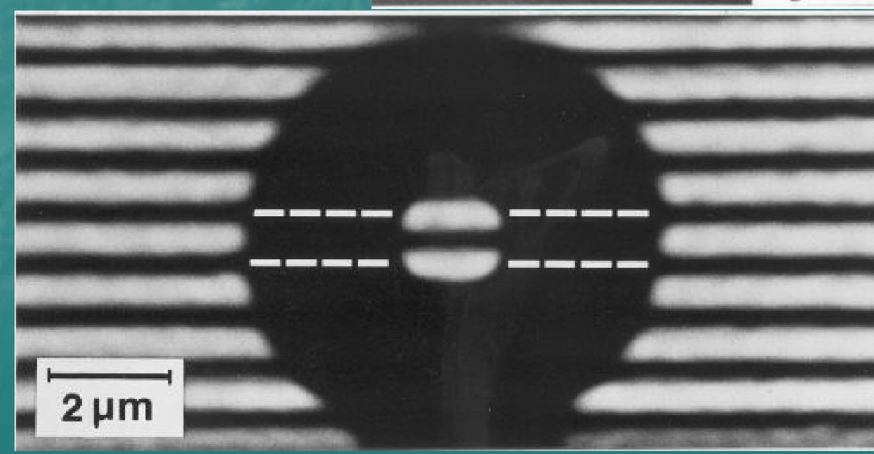
$$= \frac{\hbar}{e}\oint_{\mathrm{loop}} \mathbf{A}(\mathbf{r})\cdot d\mathbf{r} = \frac{\hbar}{e}\int \mathbf{B}(\mathbf{r})\cdot d\mathbf{S}$$

$$= 2\pi\frac{\phi}{\phi_0} \qquad \phi_0 = \frac{h}{e}$$

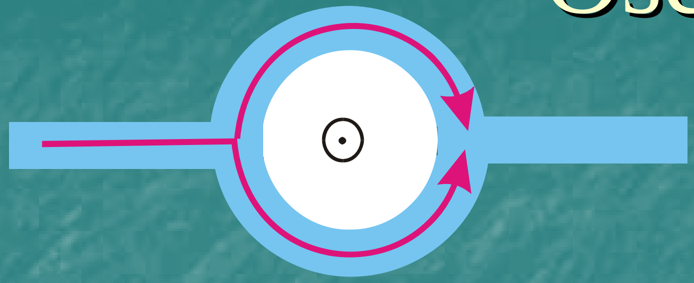


出典確認中 ‡



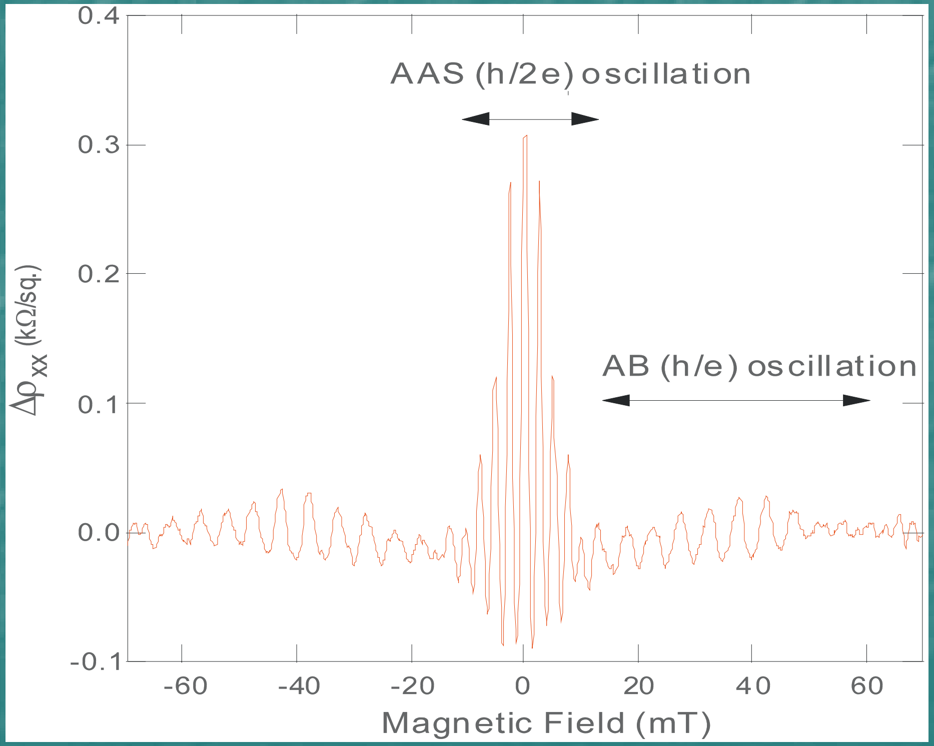


Aharonov-Bohm Oscillation (AB Oscillation)



$$\Delta\theta = \frac{\hbar}{e}\int_{L}\mathbf{A}(\mathbf{r})\cdot d\mathbf{r} - \frac{\hbar}{e}\int_{R}\mathbf{A}(\mathbf{r})\cdot d\mathbf{r}$$
$$= \frac{\hbar}{e}\oint_{\text{loop}}\mathbf{A}(\mathbf{r})\cdot d\mathbf{r} = \frac{\hbar}{e}\int\mathbf{B}(\mathbf{r})\cdot d\mathbf{S}$$
$$= 2\pi\frac{\phi}{\phi_0} \qquad \phi_0 = \frac{h}{e}$$

Magnetic flux quantum



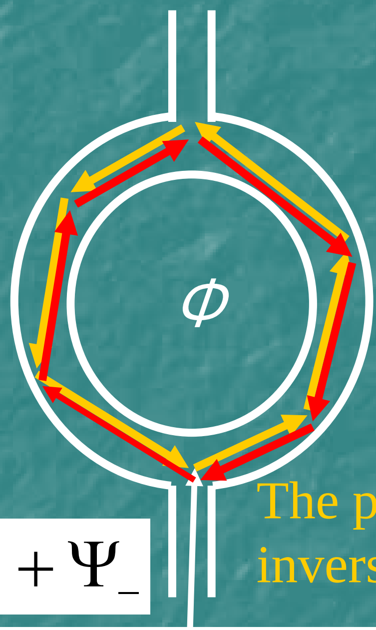
$$\phi_0 = \frac{e}{h} = 4.14\times10^{-15}\,\text{Wb}$$

h/*e* oscillation and *h*/2*e* oscillation

The above shows the interference of electron waves that pass through the paths in both sides of the ring. Electric resistance periodically oscillates for the ring penetrating magnetic flux Φ.

Electron Locality as Quantum Interference

h/2e Oscillation



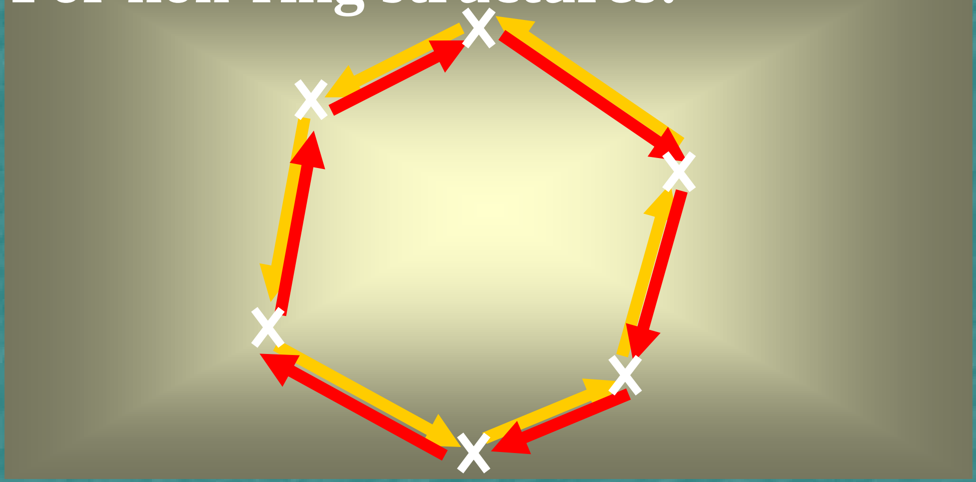
$$\Psi = \Psi_+ + \Psi_-$$

$$|\Psi|^2 = |\Psi_+ + \Psi_-|^2$$
$$= |\Psi_+|^2 + |\Psi_-|^2 + \underbrace{\Psi_+ \Psi_-^* + \Psi_+^* \Psi_-}_{\text{Quantum interference term}}$$

Quantum interference term

The pair is in a relationship of inverse symmetry of time.

For non-ring structures:



$$\sigma_{2D} = \frac{ne^2\tau}{m} - \frac{P}{\pi}\frac{e^2}{h}\ln\frac{T}{T_0}$$

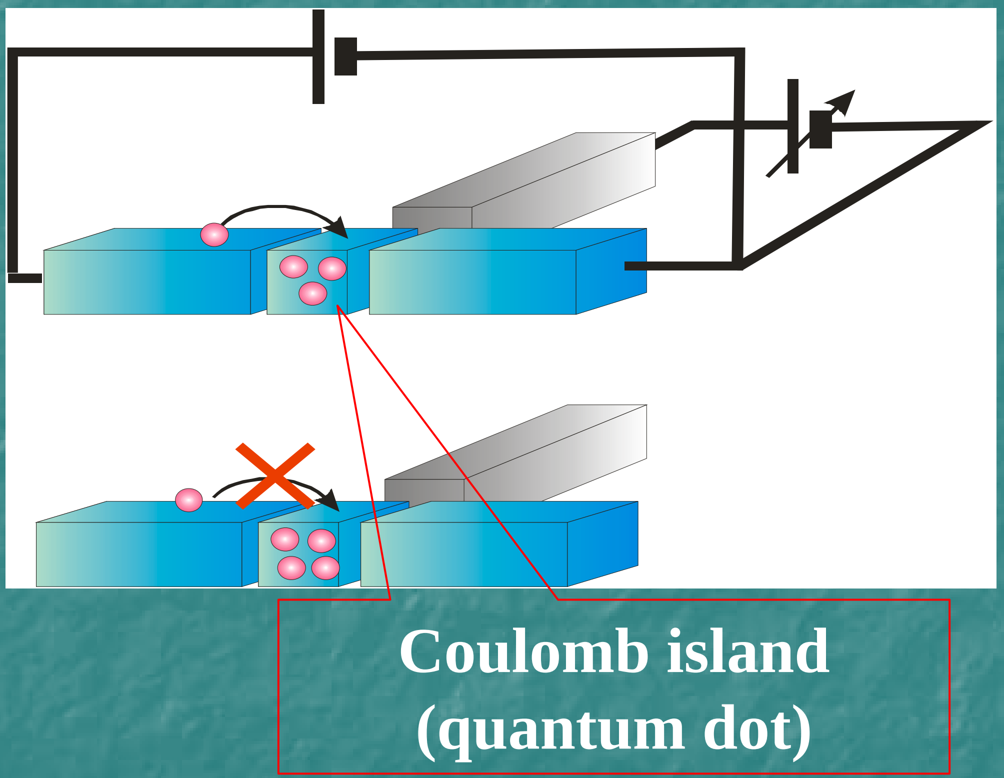
Classical conductivity

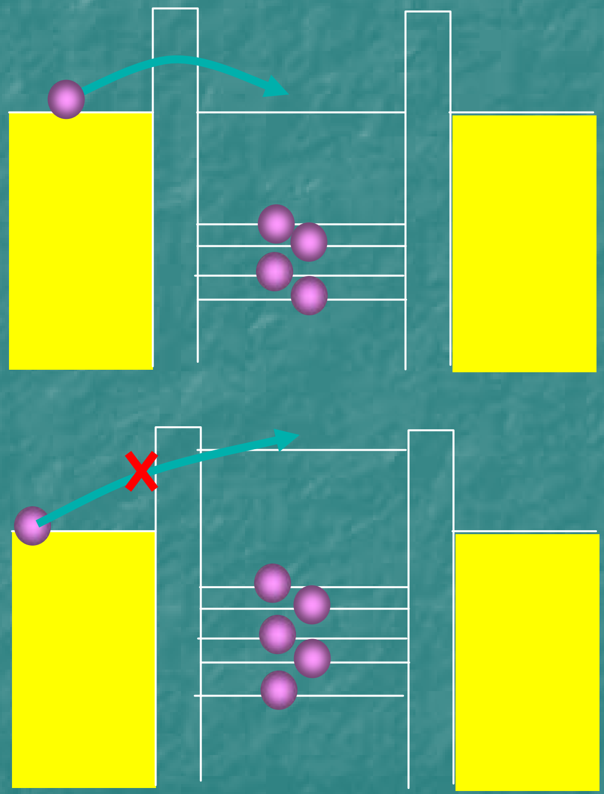
Quantum correction

Conductance fluctuation~e^2/h

Mono-electron Tunnel Effect

Micro tunnel junction

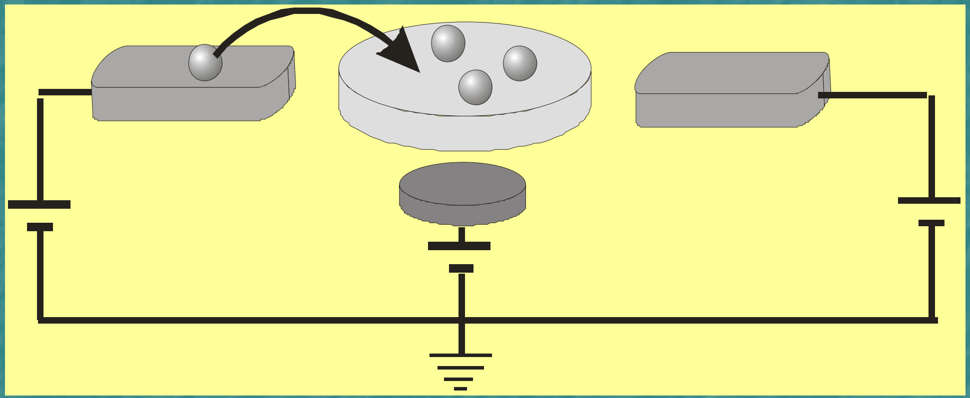


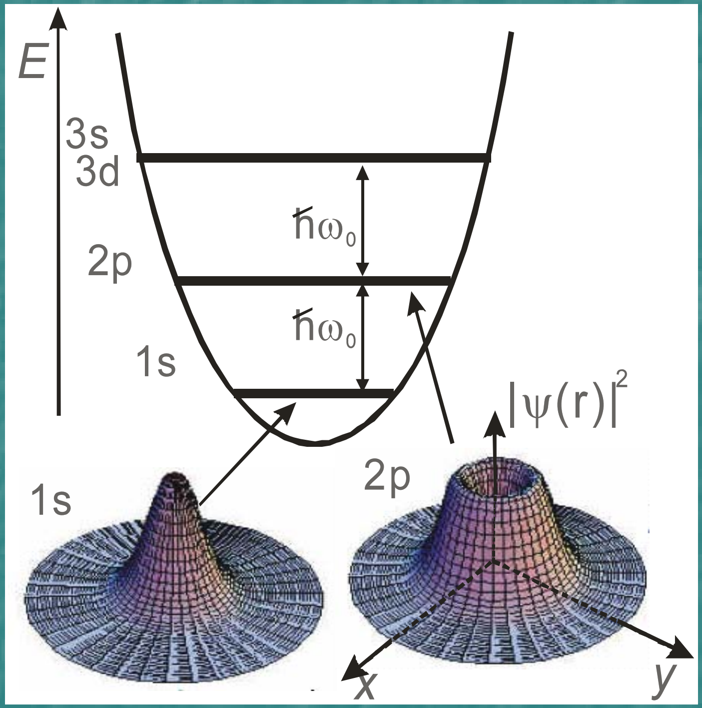


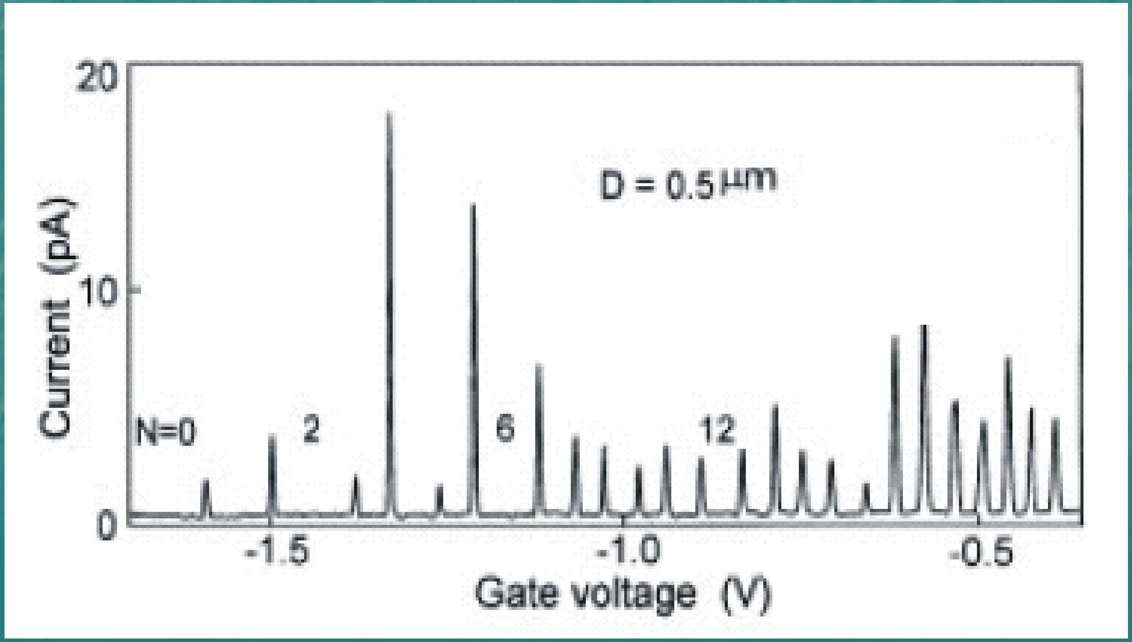
Electrostatic potential of island is increased due to tunneling of a single electron.
⇒ the following electron cannot go in.

Coulomb blockade

Quantum Dot (Artificial Atom)



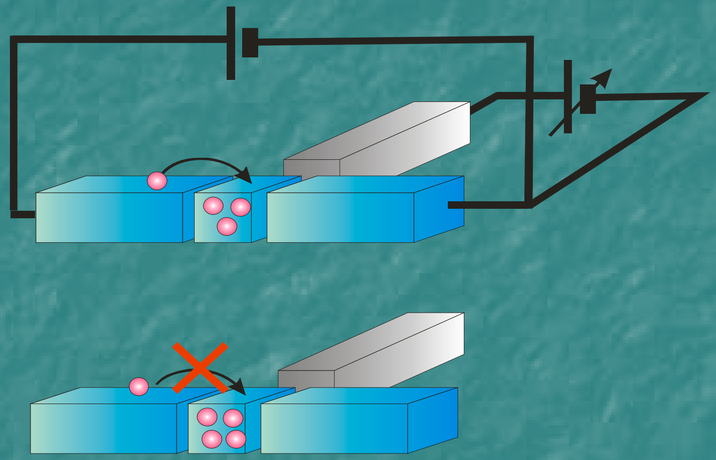


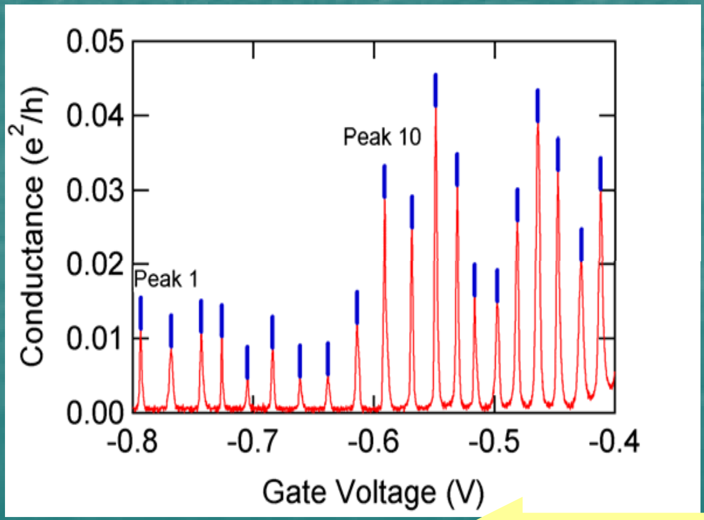


Prof. Tarutya, Seigo., School of Science, University of Tokyo

Quantum Physics in Mesoscopic System

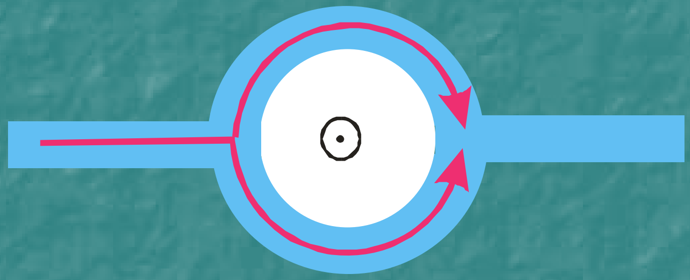
Mono-electron tunnel effect

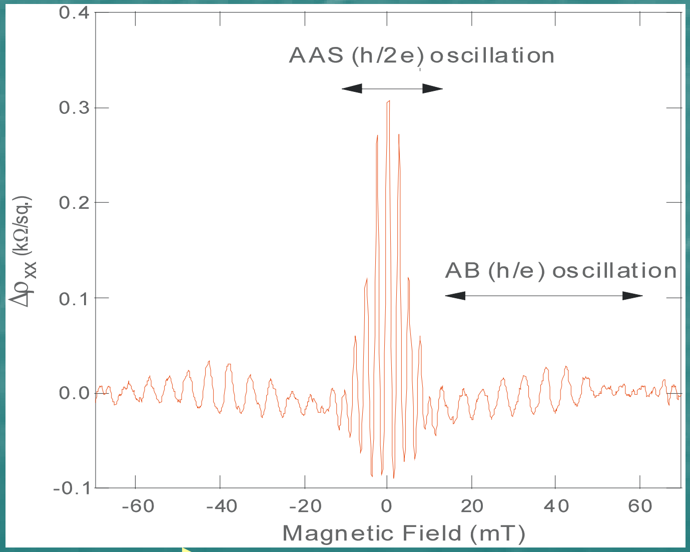




Wave-particle duality of electrons

Quantum interference effect





Particle properties

Wave properties

Summary

Mesoscopic physics

Artificially-designed system
＋state-of-the-art observation
and measuring methods

Visible quantum mechanics:
wave properties quantum interference effect
particle properties mono-electron tunnel effect

Magic number $e^2/h = (25.813 \mathrm{kW})^{-1}$

Incorporation of fundamental physics and high-tech