Quantum Mechanics 3

22. August. 2006

2005 Winter Session

Hatsugai

Contents

PartI Scattering Theory **6**

1 Scattering Theory in One Dimention **6**

1.1 Transfer Matrix Method 6

1.1.1 Transfer Matrix for Scattering State and Bound State 6

1.1.2 The Transfer Matrix and the Scattering Matrix 12

1.2 The Green's Function and Scattering Integral Equations 16

1.3 Levinson's Theorem in One Dimension 22

2 The Scattering Theory in Three Dimention **29**

2.1 The Scattering Amplitude and the Differntial Cross Sections 29

2.2 Lippmann-Schwinger Equation and the scattering Amplitude 33

2.3 Born Approximation 35

2.4 Partial Wave Decomposition 37

2.4.1 The Schroedinger Equation in Spherical Symmetric Field . . 37

2.4.2 Phase Shift 45

2.4.3 Lograrithmic Differntiation and the Phase Shift 48

2.4.4 Jost Function and the Bound States 50

3 Time-dependent Scattering Theory **54**

3.1 Lippmann-Schwinger Equation 54

3.2 Optical Theory 59

PartII Relativistic Quantum Mechanics **62**

4 Special Relativity (Classical Theory) **62**

4.1 Lorentz Transformation 63

4.2 Effects of Free Particles 67

4.3 Particle Motion in Electromagnetic Field (Lagrange Formulation) . 70

4.4 Particle Motion in Electromagnetic Field (Hamilton Formulation) . 77

5 Dirac Equations **82**

5.1 Deriving the Dirac Equation 82

5.2 Symmetry of Dirac Equation 85

5.2.1 The Lorentz Invariance 88

5.3 The Plane-wave Solutions for the Free Dirac Equation 93

5.3.1 In the Case of $m \neq 0$ 95

5.4 The Non-relativistic Limit 96

PartIII Quantum Mechanics of ManyParticle System 102

6 Second Quantization and Interaction 102
6.1 The Classical Equation of Motion for a Single Particle 102
6.2 (First) Quantization of a Single Free Particle 103
6.3 First Quantization of Many Particle Systems 105
6.4 Many-particle Quantum Mechanics and the Symmetry by Particle Switching 106
6.5 First Quantization of N-Free Particles System 108
6.6 Second Quantization 111
6.7 Operator and the Interactoin in Second Quantization Formalism . . 114

7 Single-particle States and Mean-field Approximations in Fermion Systems 118
7.1 Single-particle Orbit and Unitary Transformation of Fermi Operator 118
7.2 Total Energy of Single-particle States 120
7.3 Mean Field Equations: Hartree-Fock Equations 124

8 The Single-particle State and Mean Field Approximation in Electron Spin System 129
8.1 Hamiltonian of Many-particle System 129
8.2 Spin-orbital Function 129
8.3 The Total Energy of Single-particle States 130
8.4 The Hartree-Fock Equation in Electron Systems 131

PartIV Electronic Structure of Many-Electron Atoms 133

9 Periodic Table and One-electronic Level of Atoms 133
9.1 One-electronic Level Structure of a Hydrogen-like Atom 133
9.2 The Hamiltonian in Many-electron Atoms 143
9.3 Periodic Law of the Elements and the Shielding Effect 144

10 Electron Configurations and Multiplet Structures 147
10.1 Multiplet Terms and Perturbation Theory 147
10.2 Angular Momentum Operator, Spin-orbital Function, and Second Quantization 155
10.3 Some Concrete Examples of Multiplet Terms and the Method of Trace 158

10.3.1 $(1s)(2s)$ . . . 158
10.3.2 $(1s)(1s)$ . . . 162
10.3.3 $(1s)(2s)(3s)$ . . . 162
10.3.4 $(2p)(3p)$ . . . 163
10.3.5 $(2p)^2$ . . . 166
10.3.6 pd . . . 168
10.3.7 pds . . . 168
10.4 Electron-hole Transformation and the Multiplet $(nl)^x$ . . . 168
10.4.1 Multiplet $(nl)^x$ . . . 168
10.4.2 Electron-hole Transformation . . . 169
10.5 Hund 's Rule . . . 171
10.6 Spin-orbit Interaction . . . 172

PartV Interaction of Light and Matter 179

11 Classical Electromagnetic Field 179
11.1 Maxwell 's Equation . . . 179
11.2 The Vector Potential and Scalar Potential . . . 181
11.3 Classical Field Equations . . . 188
11.4 Field Momentum . . . 192
11.5 Angular Momentum of the Field . . . 194

12 The Interacting Particle System and Electromagnetic Field as Field Quantity 196
12.1 Lagrangian Density and Equation of Motion . . . 196
12.2 Energy-momentum Tensor and the Conservation Law . . . 198

13 Quantization of Electromagnetic Field and the Charged Particles 200
13.1 Hamiltonian . . . 202
13.2 Momentum . . . 203

14 Interaction of Electromagnetic Field with Matter 204
14.1 Fermi 's Golden Rule . . . 206
14.2 Transition Matrix Elements and Dipole Transition . . . 208
14.3 Light Emission . . . 212
14.4 Light Absorption . . . 213

PartVI Appendix 214

A Separation of Variables for Hel,holtz Equation in Polar Coordinates **214**

B Spherical Function **215**

B.1 Legendre Differential Equation 215

B.2 Associated Legendre Differential Equation 216

B.3 Spherical Function 216

C Spherical Bessel Function **217**

C.1 Spherical Bessel Function 217