Chapter 5

Schroedinger Equation

5.1 Deduction of Schroedinger Equation

Classical particles in the free space has a relation $E = p^2/2m$. By $\Delta x \to 0$, we can deduce the same equation from the eigen value in the previous section. Let $\Delta x \to 0$ by keeping the relations $A = (\hbar/\Delta x)^2/2m$ and $E_0 = 2A$, then the eigen value E vs. p reaches the classical relation. The equation of motion becomes

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2}.$$

Replace $\psi(x,t)$ into $\langle \underline{x}|\,\psi(t)\rangle$. When the position potential $V(x)$ varies, the E_0 receives an additional variable, and the equation will be modified as

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t).$$

This is "Schroedinger equation".

5.2 Solution of Schroedinger in One-Dimesional Space

Suppose the stationary solultion $\psi(x,t) = \Psi(x)\exp(Et/i\hbar)$, Schroedinger equation becomes

$$E\Psi(x) = -\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + V(x)\Psi(x).$$

From the equation, the second derivative is

$$\frac{d^2\Psi(x)}{dx^2} = \frac{2m}{\hbar^2}[V(x) - E]\Psi(x).$$

This is an indication that the curvature of $\Psi(x)$ can be calculated if we knew the values $V(x), E$ and $\Psi(x)$. The solution takes an exponential function when $V(x) > E$, which is like the sinusoidal function when $V(x) < E$. In getting a meaningful solution that is finite in all space, it should be expanding to the exponetial function in a region $x \to -\infty$ where $V(x) \to 0$, and should be converging to the exponential function in a region $x \to \infty$ where $V(x) \to 0$. Calculate the function from the left side to the right side, the value of E should be limited to the certain values in order to get the converging exponential function in the right region.

These E values corresponds to eigen values of the equation, therefore, we can conclude that the wave function (eigen state) oscillates more in the region of $V(x) < E$ with the higher E.