April 23 Lecture Schdule

Inverse trigonometric function: ([1] p.12-15,p.28-29, [2] p.35-37, p.58-59, [3] p.87, p.115, p.167-170.)

We call $(a,b) = \{x \in \mathbf{R} | a < x < b\}$an open interval, and $[a,b] = \{x \in \mathbf{R} | a \leq x \leq b\}$ a closed interval.

A rule that defines a single real number y for each $a \leq x \leq b$ is called a function defined on the closed interval $[a,b]$.

Let $f(x)$ be the strictly monotone increasing continuous function defined on the closed interval $[a,b]$, and let $c = f(a)$,and $d = f(b)$.

Now, the strictly monotone increasing continuous function $g(x)$ defined on $[c,d]$ gives values which satisfy $y = g(x) \Leftrightarrow x = f(y)$ for the arbitrary $x \in [c,d]$, and $y \in [a,b]$. Such function is called the inverse function of f.

If $f(x)$ is differentiable then, $g(x)$ is also differentiable so that $g'(x) = 1/f'(g(x))$ can be given.

[Proof of the continuity of g] will be discussed next week.

Given $f(x) = e^x$,and $(a,b) = \mathbf{R}$, we can write $(c,d) = (0,\infty)$, $g(x) = \log x$.

Given $f(x) = \sin x$,and $[a,b] = [-\pi/2, \pi/2]$, we can write $[c,d] = [-1,1]$, $g(x) = \text{Arcsin}x$, where Arcsin$x$ is the odd function.

$$\text{Arcsin}0 = 0, \text{Arcsin}\frac{1}{2} = \frac{\pi}{6}, \text{Arcsin}\frac{1}{\sqrt{2}} = \frac{\pi}{4}, \text{Arcsin}\frac{\sqrt{2}}{2} = \frac{\pi}{3}, \text{Arcsin}1 = \frac{\pi}{2}.$$

$$\text{Arcsin}'x = \frac{1}{\sin'(\text{Arcsin}x)} = \frac{1}{\cos(\text{Arcsin}x)} = \frac{1}{\sqrt{1-x^2}}.$$

In other words, the above describes the arc length formed by a point $P(x, \sqrt{1-x^2})$ which connects to a point $A(0,1)$ found on the circle of radius 1 with a center at the origin.

$$\begin{aligned}\text{Arcsin}'x &= \int_0^x \sqrt{1 + \left(\frac{d}{dx}\sqrt{1-x^2}\right)^2} dx = \int_0^x \sqrt{1 + \left(\frac{x}{\sqrt{1-x^2}}\right)^2} dx \\ &= \int_0^x \frac{1}{\sqrt{1-x^2}} dx\end{aligned}$$

If $f(x) = \tan x$, and $(a,b) = (-\pi/2, \pi/2)$, we can write $(c,d) = \mathbf{R}$, $g(x) = \text{Arctan}x$, where Arctan$x$ is the odd function.

$$\text{Arctan}0 = 0, \text{Arctan}\frac{1}{\sqrt{3}} = \frac{\pi}{6}, \text{Arctan}1 = \frac{\pi}{4}, \text{Arctan}\sqrt{3} = \frac{\pi}{3},$$

$$\lim_{x \to \infty} \text{Arctan}x = \frac{\pi}{2}.$$

$$\text{Arctan}'x = \frac{1}{\tan'(\text{Arctan}x)} = \cos^2(\text{Arctan}x) = \frac{1}{1+x^2}.$$

$$\lim_{x\to\infty} x(\frac{\pi}{2} - \mathrm{Arctan} x) = \lim_{y\to\frac{\pi}{2}-0} \tan y(\frac{\pi}{2} - y) = \lim_{z\to 0+0} \frac{\cos z}{\sin z} z = 1.$$

In other words, the above describes the length of the arc AQ wherein A is the point $(0,1)$ and Q is the intersection point formed between a circle having the radius 1 with a center at origin O and a line OP where P is the point $(x,1)$.

Coordinates for Q are given by $\left(\frac{x}{\sqrt{1+x^2}}, \frac{1}{\sqrt{1+x^2}}\right)$.

$$\begin{aligned}
\mathrm{Arctan}' x &= \int_0^x \sqrt{\left(\frac{d}{dx}\frac{x}{\sqrt{1+x^2}}\right)^2 + \left(\frac{d}{dx}\frac{1}{\sqrt{1+x^2}}\right)^2} dx \\
&= = \int_0^x \sqrt{\left(\frac{1}{\sqrt{1+x^2}} - \frac{x^2}{\sqrt{1+x^2}^3}\right)^2 + \left(-\frac{x}{\sqrt{1+x^2}^3}\right)^2} dx \\
&= \int_0^x \frac{1}{1+x^2} dx
\end{aligned}$$