April 16 Lecture Schedule

Webpage: http://www.ms.u-tokyo.ac.jp/∼t-saito/ce.html

Exercises： Gocho Research Associate Tuesday, 4th Period, Room 574

Review and additional discussions of previous lecture:([1] p.6-7, p.182 Theorem3, [2] p.12-13, 22-23, [3] p.43-44.)

A series $\sum_{n=0}^{\infty}\frac{1}{n!}$ converges. The sum of the series equals to $e=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$. The partial sum of $s_m,\ m=0,1,2,\ldots$ is given by $|e-s_m|\le 1/2^{m-1}$.

The restatement of the continuity of real numbers: ([1] p.6 Theorem3, [2] p.12 Theorem 1.1, [3] p.37 Theorem 1.20.) A sequence which is monotonically increasing and bounded above is convergent.

Absolute convergence:([1] p.188-189, [2] p.152 Theorem 5.5, [3] p.42 Theorem 1.21.) If $\sum_{n=0}^{\infty}|a_n|$ is convergent then

$\sum_{n=0}^{\infty}a_n$ must be convergent. In such a case, $\sum_{n=0}^{\infty}a_n$ is known as absolute convergence.

Method of majorant series: ([1] p.184 Theorem 5, [3] p.42 Theorem 1.22) If $|a_n|\le b_n, n=0,1,2,\ldots$ and $\sum_{n=0}^{\infty}b_n$ are convergent then, $\sum_{n=0}^{\infty}a_n$ is absolutely convergent.

Examples: ([1] p.192 Problem 10, [3] p.43 Example 1.3) The series $\sum_{n=0}^{\infty}\frac{x^n}{n!}$ is absolutely convergent for the arbitrary real number x.

We take a natural number which satisfies $M\ge 2|x|$. If $n\ge M$ then we can write $\left|\frac{x^n}{n!}\right|\le\frac{|x|^M}{M!}\frac{1}{2^{M-n}}$.

Note that$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$ can be given.

We use this to give $\frac{e^x}{x^n}\ge\frac{x}{(n+1)!}\to\infty$ (where $x\to\infty$) for the natural number $n\ge 0$

(cf. [1] p.42-43)

$$\begin{aligned}|e-s_m| &= \sum_{n=m+1}^{\infty}\frac{1}{n!}\le\frac{1}{(m+1)!}\sum_{n=0}^{\infty}\frac{1}{(m+2)^n}\\ &\le \frac{1}{(m+1)!}\frac{1}{1-\frac{1}{m+2}}=\frac{1}{(m+1)!}\frac{m+2}{m+1}.\end{aligned}$$

Inverse trigonometric function: ([1] p.12-15,p.28-29, [2] p.35-37, p.58-59, [3] p.87, p.115, p.167-170.)