October 29 Lecture Schedule

If $f(x,y)$ is partial differentiable at each point then partial derivative $f(x,y)$ $\dfrac{\partial f}{\partial x}(x,y)$, $\dfrac{\partial f}{\partial y}(x,y)$ can be defined.

Example $\dfrac{\partial xy}{\partial y}(x,y) = x$

Since it is quite difficult to prove that the function is total differentiable by using the definition above, we use a simple way of testing.

The function is total differentiable if the partial derivative is continuous.

[Proof]

$$\begin{aligned} & f(x,y) - (f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)) \\ = \; & (f(x,b) - f(a,b) - f_x(a,b)(x-a)) + (f(x,y) - f(x,b) - f_y(a,b)(y-b)) \end{aligned}$$

For the first term of the equation, we use the definition of partial differential and write as

$$\frac{|f(x,b) - f(a,b) - f_x(a,b)(x-a)|}{\sqrt{(x-a)^2 + (y-b)^2}} \leq \frac{|f(x,b) - f(a,b) - f_x(a,b)(x-a)|}{|x-a|} \to 0$$

For the second term, we use the mean value theorem and write as

$$f(x,y) - f(x,b) - f_y(a,b)(y-b) = (f_y(x,t) - f_y(a,b))(y-b)$$

.

t satisfying above exists between y and b. If the partial derivative $f_y(x,y)$ is continuous at $(x,y) = (a,b)$ then $(x,t) \to (a,b)$ when $(x,y) \to (a,b)$ such that

$$\frac{|(f_y(x,t) - f_y(a,b))(y-b)|}{\sqrt{(x-a)^2 + (y^b)^2}} \leq |f_y(x,t) - f_y(a,b)| \to 0.$$