October 08 Lecture Schedule

Differentiation of multivariable functions:

・Partial differentiation, differentiation of composite function and its application

・Change of the order of total and partial differentiation

・Taylor 's theorem

・Local maximum and minimum

Multiple integrals

・Definition

・Iterated integral

・Application (area, volume, center of gravity, and etc.)

・Change of variables formula and its application

Function of two variables: ([Nagase, Ashino] 7.1, [Kaneko] II 6.1, [Kodaira] II §6.1) It is the rule which makes the real number $f(x, y)$ corresponds to the pair of the real number (x, y).

Example. $f(x, y) = x + y, xy, e^{-x^2-y^2}, \ldots$

Note that the equations which are not given here can be found in the expressions for the single variable case.

The domain is not necessary the entire domain $\mathbb{R}^2 = \{(x, y) | x, y \in \mathbb{R}\}$.

Example $f(x, y) = \frac{x}{y}, x^y = e^{y \log x}, \ldots$

Visualization of two variables function:

First, we assume that the domain $\mathbb{R}^2$ is identical with a set of all points in a plane.

Graphs of the function

$\{(x, y, z) | (x, y) \in \text{domain}, z = f(x, y)\}$ determines the subset of the space $\mathbb{R}^3$, and we need to illustrate that.

Example: upper half of the spherical surface $z = \sqrt{1 - (x^2 + y^2)}$

Plane $z = ax + by + c$

Position vector of the point on $z = ax + by + c$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = x \begin{pmatrix} 1 \\ 0 \\ a \end{pmatrix} + y \begin{pmatrix} 0 \\ 1 \\ b \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ c \end{pmatrix}$

In a general case: draw many graphs of $z = f(x, b), z = f(a, y)$.

Limits of the two variables function: $\lim_{(x,y) \to (a,b)} f(x, y)$, which has a different meaning from $\lim_{x\to a}\left(\lim_{y\to b} f(x,y)\right)$ and $\lim_{y\to b}\left(\lim_{x\to a} f(x,y)\right)$

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Example： $\lim_{x\to 0}\left(\lim_{y\to 0}\frac{xy}{x^2+y^2}\right)=0$, $\lim_{y\to 0}\left(\lim_{x\to 0}\frac{x^2}{x^2+y^2}\right)=0$.

Yet, if we let $x=y$ and $x\to 0$, we may write $\lim_{x\to 0}\frac{x^2}{2x^2}=\frac{1}{2}$.

If we take $x=r\cos\theta$,and $y=r\sin\theta$ then we can write $\frac{xy}{x^2+y^2}=\frac{\sin 2\theta}{2}$。

Note that $\lim_{(x,y)\to(a,b)} f(x,y)=A$ is the function which satisfies $\lim_{r\to+0} g(r)=0$, and if $0<(x-a)^2+(y-b)^2\leq r^2$ then we can take the funcion that satisfies $|f(x,y)-A|\leq g(r)$. (Using ε-δ proof, r represents δ,while $g(r)$ represents ε.)

Example：If $f(x,y)=xy$, $|xy-ab|=|(x-a)(y-b)+(x-a)b+a(y-b)|\leq |x-a||y-b|+|x-a||b|+|y-b||a|\leq r^2+(|a|+|b|)r(=g(r))$