April 9 Lecture Schedule
Syllabus

1. Real Numbers, Limits, Series, Continuity
2. Differential Calculus, Taylor Expansion, Power Series
3. Integral Calculus, Definite Integral, Rational Function, Improper Integral

In the latter half of the course: Multivariable

1. Bibunsekibun Gairon by Takahashi and Kato, edited by Koshi, published by Saiensu-sha. (Minimum necessary)
2. Surikei-notameno Kisoto-oyo: Bibunsekibun I, II by Kaneko, published by Saiensu-sha.
3. Kaiseki-nyumon I, II by Kodaira, published by Iwanami Shoten. (For advanced learner)

Lecture Content ([1] p.6-7, 179, 182 Theorem 3. [2]p. 12-13, 22-23。 [3] p.43-44)

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 2.7182818\ldots.$$

Why does convergence take place? How can the value be obtained?

$$e' = \sum_{n=0}^{\infty}\frac{1}{n!}(= e).$$

For a sequence $a_n, n = 0, 1, 2, \ldots$, we call

$$\sum_{n=0}^{\infty} a_n$$

a series. If a sequence of partial sums $s_m = \sum_{n=0}^{m} a_n$converges, the limit $\lim_{m\to\infty} s_m$ is called the sum of a series $\sum_{n=0}^{\infty} a_n$ and expressed as $\sum_{n=0}^{\infty} a_n$

Example : a geometric series $\sum_{n=0}^{\infty} a^n$ converges at $|a| < 1$. The sum of the series can be determined as $\dfrac{1}{1-a}$.

Convergence condition

Continuity of real numbers: a necessary and sufficient condition for a series of positive terms to converge is that a sequence of the partial sums of the series to be bounded above.

The series $\sum_{n=0}^{\infty}\frac{1}{n!}$ satisfies the above convergence condition. Indeed, we can write,

$$\frac{1}{n!} \le \frac{1}{2^{n-1}}. \tag{1}$$

Thus,$s_m \leq 3$.

Continuity of real numbers (restatement): a monotonically increasing sequence that is bounded above is convergent. The real numbers?the approximate values can be derived as precise as we choose. The inequality (1) gives

$$|e' - s_m| \leq \frac{1}{2^{m-1}}.$$

Problem: Find the value of m in order to show $e' = 2.7182818\ldots$. Find s_m for obtained m to verify the equation above.

Proof of $e' = e$:

$$\begin{aligned} |e' - \left(1 + \frac{1}{m}\right)^m| &\leq |e' - s_m| + \left| s_m - \left(1 + \frac{1}{m}\right)^m \right| \\ &\leq \frac{1}{2^{m-1}} + \sum_{n=0}^{m} \frac{1}{n!} \left(1 - \left(1 - \frac{1}{m}\right) \cdots \left(1 - \frac{n-1}{m}\right)\right). \end{aligned}$$