	Semiconducting oxides
Physics of Transition Metal Oxides Lecture 6 Semiconducting oxides	Remember the carrier concentration calculations in semiconductors. The conclusions were that the Fermi level would be approximately in the middle of the gap $E_{\rm F}\approx \frac{E_c+E_v}{2},$ and we could estimate the <i>intrinsic carrier concentration</i> $n=p=\sqrt{N_cN_v}\exp(-E_g/2kT).$ These conditions rarely apply to oxides, except for very pure stoichiometric samples or very high temperatures. In most cases the carrier concentration is determined by defects or impurities. n -type behavior $(n \gg p)$ is very common, as this is caused by even a slight reduction of an insulator (TiO_{2-x}) or by substitutional doping $(\text{SrNb}_x\text{Ti}_{1-x}\text{O}_3)$.
1	2
The charge of the extra carriers is neutralized by lattice defects, which must carry	If we write the donor ionization process as
a positive net charge. It is therefore also possible to produce a bound impurity	$D = D^+ + e^-$
state for each electron. The energy that is needed to excite an electron from this	and remmeber the law of mass action
bound state to the conduction band is the <i>donor ionization energy</i> E_d .	$np pprox N_c N_v \mathrm{e}^{rac{-E_g}{k_B T}},$
Figure renoved due to corpenjate restrictions	we can calculate the carrier concentration
	$n[D^+] = [D]N_c \exp(-E_d/kT),$
The typical value for E_d is 0.1 to 0.5 eV in oxides. These numbers are generally larger than for simple semiconductors, due to the larger energy gap in oxides	where [D] and $[D^+]$ athe the concentrations of neutral and ionized donors. We have also replaced E_g with E_d . If these donors are the only source of carriers, we must have
	$n = [D^+].$
	and we get the carrier concentration as
Figure removed due to	$n \propto \exp(-E_d/2kT).$
copyright restrictions	The Fermi level is now half-way between the donor level and the conduction band bottom.
Grosso00 p.483	<i>p</i> -type semiconductors work the same way. Electrons can be removed by additional oxidation of a stoichiometric compound or by doping. For example, $Li_xNi_{1-x}O$ is a <i>p</i> -type semiconductor.

The sim in the la metal va <i>compens</i> In an <i>n</i> - number This wo where the pinned co	ple description of ttice. This is no cancies may also sation, trapping type material, st of ionized donor uld give a donor ne activation end	only ap ot true o have the ho trong o rs, ever conce ergy is or level	pplies if there for oxides, in oxygen vacance les produced for compensation if no free ca $n \ll [D^+]$ ntration $n \propto \exp(-E_d)$ E_d , not $E_d/2$ at an energy	only a single ty n general. A <i>p</i> -ty cies. It is thus po by the majority of by acceptor centriers are present $[l_l/kT]$ 2 and the Fermi E_d below the co	pe of ype cc ossible defects ters w ; and level onduct	defect present omopound with to have charge s. vill give a large would thus be ion band edge.	Figure removed due to copyright restrictions	The low-temperature region is known as the <i>freeze-out range</i> At higher temperature <i>all</i> donors or acceptors are ionized and the carrier concentration does not depend on temperature. This is the <i>saturation</i> <i>range</i> . At very high temperatures the majority of carriers are intrinsic (i.e. thermally excited from the valence band). This is the <i>intrinsic</i> <i>range</i> .
T I							Singleton01 p 61]
The con present known. This disc enough)	nplicated structu and the exact ty cussion applied to for only a small	ore of ore of ora cas numb	oxides means compensating the where the the er of donors/	that strong cor I, or even majori emperature is lov acceptors to be i	npensa ty defe v enou ionized	ation is usually ects is not well high (or E_d large d. 5	In oxides it is often difficult to mea tion or mobility. Samples are usual Obtaining good contact between a transport properties may be domina	sure conductivity and thus carrier concentra- ly not single crystals, but powders or grains. sample and electrode is difficult. Even then, ted by grain boundaries. 6
<u> </u>		C	a secolar de la secolar					
Some co	onductivity numb	oers for	oxides:				The conductivity of an oxide is a su	m over all carrier types
Some co	Compound	ers for Type	oxides:	Effective mass	Ea		The conductivity of an oxide is a su $\sigma =$	m over all carrier types $e^{ine\mu_e} + pe\mu_h,$
Some co	Compound	Type	Mobility cm ² V ⁻¹ s ⁻¹	Effective mass m^*/m_0	E_a	E_{μ}	The conductivity of an oxide is a su $\sigma =$ where μ_e and μ_h are the carrier mob	m over all carrier types $re\mu_e + pe\mu_h,$ ilities, defined as their average drift velocity in
Some co	Compound TiO _{2-x}	Type	Mobility $cm^2V^{-1}s^{-1}$ 0.1 0.1	Effective mass m^*/m_0 20	<i>E</i> _a 0.03	<i>Ε</i> μ 0	The conductivity of an oxide is a su $\sigma =$ where μ_e and μ_h are the carrier mob an electric field of unit strength	m over all carrier types $ne\mu_e + pe\mu_h$, ilities, defined as their average drift velocity in
Some co	Compound TiO _{2-x} Ti _{1-x} Nb _x O ₂ SrTiO _{3-x}	Type	Mobility $cm^2V^{-1}s^{-1}$ 0.1 0.1 3	Effective mass m^*/m_0 20 - 12	<i>Ea</i> 0.03 0.14 0	$ \begin{array}{c} \hline E_{\mu}\\ \hline 0\\ 0\\ 0\\ \hline \end{array} $	The conductivity of an oxide is a su $\sigma =$ where μ_e and μ_h are the carrier mobian electric field of unit strength	m over all carrier types $= ne\mu_e + pe\mu_h$, ilities, defined as their average drift velocity in
Some co	$\frac{\text{Compound}}{\text{TiO}_{2-x}}$ $\frac{\text{TiO}_{2-x}}{\text{Ti}_{1-x}\text{Nb}_x\text{O}_2}$ SrTiO_{3-x} BaTiO_{3-x}	Type n n n n	Mobility $cm^2V^{-1}s^{-1}$ 0.1 0.1 3 -	Effective mass <i>m</i> */ <i>m</i> ₀ 20 - 12 -	<i>E</i> _a 0.03 0.14 0	E_{μ} 0 0 0 0 0.15	The conductivity of an oxide is a su $\sigma =$ where μ_e and μ_h are the carrier mob an electric field of unit strength In some cases, if all dominant carrie	m over all carrier types $re\mu_e + pe\mu_h$, ilities, defined as their average drift velocity in er defects are fully ionized, and the number of
Some co	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	Type n n n n p	Mobility cm ² V ⁻¹ s ⁻¹ 0.1 0.1 3 - 0.15	Effective mass <i>m</i> */ <i>m</i> ₀ 20 - 12 - 17	<i>E</i> _a 0.03 0.14 0 - 0.1	E_{μ} 0 0 0.15 0	The conductivity of an oxide is a su $\sigma =$ where μ_e and μ_h are the carrier mobian electric field of unit strength In some cases, if all dominant carrier defects is known from the chemical c	m over all carrier types $re\mu_e + pe\mu_h$, ilities, defined as their average drift velocity in the defects are fully ionized, and the number of omposition, mobility can be directly calculated
Some co	Compound TiO_{2-x} $Ti_{1-x}Nb_xO_2$ $SrTiO_{3-x}$ $BaTiO_{3-x}$ $La_{1-x}Sr_xVO_3$ $Li_xV_2O_5$	Type n n n p n	Mobility cm ² V ⁻¹ s ⁻¹ 0.1 0.1 3 - 0.15 0.03	Effective mass <i>m</i> */ <i>m</i> ₀ 20 - 12 - 17 -	<i>E_a</i> 0.03 0.14 0 - 0.1 0.1	E_{μ} 0 0 0.15 0 0.15	The conductivity of an oxide is a su $\sigma =$ where μ_e and μ_h are the carrier mobility an electric field of unit strength In some cases, if all dominant carried defects is known from the chemical of from the conductivity.	m over all carrier types $re\mu_e + pe\mu_h$, ilities, defined as their average drift velocity in er defects are fully ionized, and the number of omposition, mobility can be directly calculated
Some co	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	Type n n n p n p	r oxides: Mobility $cm^2 V^{-1} s^{-1}$ 0.1 0.1 0.1 0.15 0.03 - 10-5	Effective mass m*/m ₀ 20 - 12 - 17 - -	<i>E_a</i> 0.03 0.14 0 - 0.1 0.1 0	$ \begin{array}{c} E_{\mu} \\ 0 \\ 0 \\ 0 \\ 0.15 \\ 0.15 \\ 0.11 \\ 0.4 \\ \end{array} $	The conductivity of an oxide is a su $\sigma =$ where μ_e and μ_h are the carrier mobility an electric field of unit strength In some cases, if all dominant carried defects is known from the chemical of from the conductivity.	m over all carrier types $= ne\mu_e + pe\mu_h$, ilities, defined as their average drift velocity in er defects are fully ionized, and the number of omposition, mobility can be directly calculated
Some co	Compound TiO _{2-x} Ti _{1-x} Nb _x O ₂ SrTiO _{3-x} BaTiO _{3-x} La _{1-x} Sr _x VO ₃ Li _x V ₂ O ₅ La _{1-x} Sr _x CrO ₃ Mn _{1-x} O	Type n n n p n p p	r oxides: Mobility $cm^2V^{-1}s^{-1}$ 0.1 0.1 3 - 0.15 0.03 - 10^{-5} 10	Effective mass m*/m ₀ 20 - 12 - 17 - - -	E_a 0.03 0.14 0 - 0.1 0.1 0.5 1.5	$ \begin{array}{c} E_{\mu} \\ 0 \\ 0 \\ 0.15 \\ 0.15 \\ 0.11 \\ 0.4 \\ - \end{array} $	The conductivity of an oxide is a su $\sigma =$ where μ_e and μ_h are the carrier mobility an electric field of unit strength In some cases, if all dominant carrier defects is known from the chemical con- from the conductivity. The temperature dependence of con-	m over all carrier types $re\mu_e + pe\mu_h$, ilities, defined as their average drift velocity in er defects are fully ionized, and the number of omposition, mobility can be directly calculated ductivity often follows the <i>Arrhenius equation</i> ,
Some co	$\frac{\text{TiO}_{2-x}}{\text{Ti}_{1-x}\text{Nb}_x\text{O}_2}$ $\frac{\text{TiO}_{3-x}}{\text{BaTiO}_{3-x}}$ $\frac{\text{BaTiO}_{3-x}}{\text{La}_{1-x}\text{Sr}_x\text{VO}_3}$ $\frac{\text{Li}_x\text{V}_2\text{O}_5}{\text{La}_{1-x}\text{Sr}_x\text{CrO}_3}$ $\frac{\text{Mn}_{1-x}\text{O}}{\text{Li}_x\text{Mn}_{1-x}\text{O}}$	Type n n n p n p n p n p	Mobility cm ² V ⁻¹ s ⁻¹ 0.1 0.1 3 - 0.15 0.03 - 10 ⁻⁵ 10 10 ⁻⁵	Effective mass m*/m ₀ 20 - 12 - 17 - - - - - - -	E_a 0.03 0.14 0 - 0.1 0.1 0.5 1.5 0.4		The conductivity of an oxide is a su $\sigma =$ where μ_e and μ_h are the carrier mobi an electric field of unit strength In some cases, if all dominant carrie defects is known from the chemical con- from the conductivity. The temperature dependence of con- $\sigma = 0$	m over all carrier types $re\mu_e + pe\mu_h$, ilities, defined as their average drift velocity in er defects are fully ionized, and the number of omposition, mobility can be directly calculated ductivity often follows the <i>Arrhenius equation</i> , $hexp(-E_r/kT)$
Some co	Compound TiO_{2-x} $Ti_{1-x}Nb_xO_2$ $SrTiO_{3-x}$ $BaTiO_{3-x}$ $La_{1-x}Sr_xVO_3$ $Li_xV_2O_5$ $La_{1-x}Sr_xCrO_3$ $Mn_{1-x}O$ $Li_xMn_{1-x}O$	Type n n n p n p n p	Mobility cm ² V ⁻¹ s ⁻¹ 0.1 0.1 3 - 0.15 0.03 - 10 ⁻⁵ 10 10 ⁻⁵ 10 ⁻⁵ 10 ⁻²	Effective mass m*/m ₀ 20 - 12 - 17 - - - - - - - - -	$\begin{array}{c} E_a \\ 0.03 \\ 0.14 \\ 0 \\ - \\ 0.1 \\ 0.1 \\ 0 \\ 0.5 \\ 1.5 \\ 0.4 \end{array}$	$ E_{\mu} 0 0 0 0 0.15 0 0.15 0.11 0.4 - 0.3 $	The conductivity of an oxide is a su $\sigma =$ where μ_e and μ_h are the carrier mobility an electric field of unit strength In some cases, if all dominant carried defects is known from the chemical of from the conductivity. The temperature dependence of con $\sigma = A$	m over all carrier types $remetarrow ne \mu_e + pe\mu_h$, ilities, defined as their average drift velocity in arr defects are fully ionized, and the number of omposition, mobility can be directly calculated ductivity often follows the <i>Arrhenius equation</i> , $Aexp(-E_{\sigma}/kT)$.
Some co	Compound TiO_{2-x} $Ti_{1-x}Nb_xO_2$ $SrTiO_{3-x}$ $BaTiO_{3-x}$ $La_{1-x}Sr_xVO_3$ $Li_xV_2O_5$ $La_{1-x}Sr_xCrO_3$ $Mn_{1-x}O$ $Li_xMn_{1-x}O$ $Li_xCo_{1-x}O$	Type n n p p n p p p	$\begin{array}{c} \text{Mobility}\\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}\\ 0.1\\ 0.1\\ 3\\ -\\ 0.15\\ 0.03\\ -\\ 10^{-5}\\ 10\\ 10^{-5}\\ 10^{-5}\\ 10^{-2}\\ 0.4\\ \end{array}$	Effective mass m*/m ₀ 20 - 12 - 17 - - - - - - - - - - -	$\begin{array}{c} E_a \\ 0.03 \\ 0.14 \\ 0 \\ - \\ 0.1 \\ 0.1 \\ 0 \\ 0.5 \\ 1.5 \\ 0.4 \\ 0.3 \end{array}$	$ \begin{array}{c} E_{\mu} \\ 0 \\ 0 \\ 0.15 \\ 0 \\ 0.15 \\ 0.11 \\ 0.4 \\ - \\ 0.3 \\ 0? \\ \end{array} $	The conductivity of an oxide is a su $\sigma =$ where μ_e and μ_h are the carrier mobility an electric field of unit strength In some cases, if all dominant carried defects is known from the chemical of from the conductivity. The temperature dependence of con $\sigma = A$ where E_{σ} is the activation energy for	m over all carrier types $reme_e + pe\mu_h$, illities, defined as their average drift velocity in er defects are fully ionized, and the number of omposition, mobility can be directly calculated ductivity often follows the <i>Arrhenius equation</i> , $A \exp(-E_{\sigma}/kT)$.
Some co	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	pers for Type n n p p p p p p p	$\begin{array}{c} \text{Mobility} \\ \text{cm}^2 \text{V}^{-1} \text{s}^{-1} \\ 0.1 \\ 0.1 \\ 0.1 \\ 3 \\ - \\ 0.15 \\ 0.03 \\ - \\ 10^{-5} \\ 10 \\ 10^{-5} \\ 10^{-2} \\ 0.4 \\ - \end{array}$	Effective mass m*/m0 20 - 12 - 17 - - - - - - - - - - - - -	$\begin{array}{c} E_a \\ 0.03 \\ 0.14 \\ 0 \\ - \\ 0.1 \\ 0.1 \\ 0 \\ 0.5 \\ 1.5 \\ 0.4 \\ 0.3 \\ 0.3 \end{array}$	$ \begin{array}{c} E_{\mu} \\ 0 \\ 0 \\ 0 \\ 0.15 \\ 0.15 \\ 0.11 \\ 0.4 \\ - \\ 0.3 \\ 0? \\ 0 \\ 0 0 0 0 0 $	The conductivity of an oxide is a su $\sigma =$ where μ_e and μ_h are the carrier mobility an electric field of unit strength In some cases, if all dominant carrier defects is known from the chemical of from the conductivity. The temperature dependence of con $\sigma = A$ where E_{σ} is the activation energy for energy that we used in calculating t	m over all carrier types $reme_e + pe\mu_h$, ilities, defined as their average drift velocity in er defects are fully ionized, and the number of omposition, mobility can be directly calculated ductivity often follows the <i>Arrhenius equation</i> , $A \exp(-E_{\sigma}/kT)$. Fr conduction. In principle, this is the activation the carrier concentration:
Some co	Compound TiO _{2-x} Ti _{1-x} Nb _x O ₂ SrTiO _{3-x} BaTiO _{3-x} La _{1-x} Sr _x VO ₃ Li _x V ₂ O ₅ La _{1-x} Sr _x CrO ₃ Mn _{1-x} O Li _x Mn _{1-x} O Li _x Ni _{1-x} O Li _x Ni _{1-x} O Li _N D _{3-x} KTaO ₂	pers for Type n n p p p p p n p n n	Mobility cm ² V ⁻¹ s ⁻¹ 0.1 0.1 3 - 0.15 0.03 - 10 ⁻⁵ 10 10 ⁻⁵ 10 ⁻⁵ 10 ⁻² 0.4 - - 30	Effective mass m*/m ₀ 20 - 12 - 17 - - - - - - - - - - - - -	$\begin{array}{c} E_a \\ 0.03 \\ 0.14 \\ 0 \\ - \\ 0.1 \\ 0.5 \\ 1.5 \\ 0.4 \\ 0.3 \\ - \\ 0 \\ 0 \\ \end{array}$	$ \begin{array}{c} E_{\mu} \\ 0 \\ 0 \\ 0 \\ 0.15 \\ 0.15 \\ 0.11 \\ 0.4 \\ - \\ 0.3 \\ 0? \\ 0 \\ 0.4 \\ 0 \end{array} $	The conductivity of an oxide is a su $\sigma =$ where μ_e and μ_h are the carrier mobility an electric field of unit strength In some cases, if all dominant carried defects is known from the chemical of from the conductivity. The temperature dependence of con $\sigma = A$ where E_{σ} is the activation energy for energy that we used in calculating to $n \propto 6$	m over all carrier types $reme_e + pe\mu_h$, ilities, defined as their average drift velocity in er defects are fully ionized, and the number of omposition, mobility can be directly calculated ductivity often follows the <i>Arrhenius equation</i> , $A \exp(-E_{\sigma}/kT)$. Fronduction. In principle, this is the activation the carrier concentration: $\exp(-E_d/2kT)$.
Some co	Compound TiO _{2-x} Ti _{1-x} Nb _x O ₂ SrTiO _{3-x} BaTiO _{3-x} La _{1-x} Sr _x VO ₃ Li _x V ₂ O ₅ La _{1-x} Sr _x CrO ₃ Mn _{1-x} O Li _x Mn _{1-x} O Li _x Ni _{1-x} O Li _x Ni _{1-x} O Li _N bO _{3-x} KTaO _{3-x} Cox92 p.112	pers for Type n n p p p p p n n n	$\begin{array}{c} \text{Mobility} \\ \text{cm}^2 \text{V}^{-1} \text{s}^{-1} \\ 0.1 \\ 0.1 \\ 0.1 \\ 3 \\ - \\ 0.15 \\ 0.03 \\ - \\ 10^{-5} \\ 10^{-5} \\ 10^{-5} \\ 10^{-2} \\ 0.4 \\ - \\ - \\ 30 \end{array}$	Effective mass m*/m0 20 - 12 - 17 - - - - - - - - - 0.8	E_a 0.03 0.14 0 - 0.1 0.1 0.5 1.5 0.4 0.3 0.3 - 0	$ \begin{array}{c} E_{\mu} \\ 0 \\ 0 \\ 0 \\ 0.15 \\ 0.15 \\ 0.11 \\ 0.4 \\ - \\ 0.3 \\ 0? \\ 0 \\ 0.4 \\ 0 \\ \end{array} $	The conductivity of an oxide is a su $\sigma =$ where μ_e and μ_h are the carrier mobility an electric field of unit strength In some cases, if all dominant carrier defects is known from the chemical of from the conductivity. The temperature dependence of con $\sigma = A$ where E_{σ} is the activation energy for energy that we used in calculating to $n \propto 6$ In practice it is not clear if we should of compensation present, which is u	m over all carrier types $= ne\mu_e + pe\mu_h$, ilities, defined as their average drift velocity in er defects are fully ionized, and the number of omposition, mobility can be directly calculated ductivity often follows the <i>Arrhenius equation</i> , $A \exp(-E_{\sigma}/kT)$. = conduction. In principle, this is the activation he carrier concentration: $\exp(-E_d/2kT)$. use E_d or $E_d/2$, a s this depends on the degree sually not known.

Mobility may also have a sharp temperature dependence

Thermopower:

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$$\mu = B \exp(-E_{\mu}/kT),$$

where E_{μ} is the activation energy for hopping from site to site. In this case the total activation energy is the sum of ionization and hopping energies

$$E_{\sigma} = E_a + E_{\mu}$$

Possible ways to determine the mobility is to measure the Hall effect or the thermopower (Seebeck effect). We start with the Seebeck effect.

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copyright restrictions

Seebeck effect

The Seebeck effect or thermopower is used in thermocouples. What is measured is the potential difference in response to a temperature gradient. For n-type semiconductors the Seebeck coefficient is

$$\alpha = -\frac{k_B}{e} \left[\frac{E_c - E_F}{k_B T} + A \right] = -\frac{k_B}{e} [\ln(N_c/n) + A],$$

and for *p*-type

0

Here we see the thermopower and activa-

tion energy for conduction in $Co_{3-x}Fe_xO_4$. At x = 2 there is a switchover from p to

n-type conductivity.

Cox92 p.164

$$\alpha = +\frac{k_B}{e} \left[\frac{E_F - E_v}{k_B T} + A \right] = +\frac{k_B}{e} [\ln(N_v/p) + A].$$

A depends on how the carrier mobility varies with energy in a band. It is usually smaller than the first term and can be neglected.

The sign of α will tell us what type of (dominant) charge carriers we have in a material, holes or electrons.

10

For a nondegenerate semiconductor $(E_c - E_F \gg k_B T)$, the carrier concentration is

$$n = N_c \mathrm{e}^{-\frac{E_c - E_F}{k_B T}}$$

If the Fermi level is pinned at the donor level, $E_c - E_F$ is the donor ionization energy. Putting n into the thermopower expression

$$\alpha = -\frac{k_B}{e} \ln(N_c/n),$$

We can see that if we plot αe as a function of 1/T, we can get the activation energy for producing free carriers from the slope of the plot.

ence bands is given by		Here we have scaled thermopower and conductivity $(\log(\sigma))$ of <i>p</i> -type NiO plotted as a function of $1/T$. The slopes are	
$0^{19}/cm^3$	Figure removed due to convribil restrictions	equal, which shows that the conduction	
0 ¹⁹ /cm ³		tivation energy of ionizing a carrier.	
perhaps a transport measure-		then we know that there is an additional mobility barrier in addition to the ioniza-	
11	Cox92 p.165	tion barrier.	12

The densities of available states in conduction or vale

$$N_c(T) = 2.5 \left(\frac{m_c^*}{m}\right)^{3/2} \left(\frac{T}{300\text{K}}\right)^{3/2} \times 10^{19}/\text{cm}^3$$
$$N_v(T) = 2.5 \left(\frac{m_v^*}{m}\right)^{3/2} \left(\frac{T}{300\text{K}}\right)^{3/2} \times 10^{19}/\text{cm}^3$$

If we have an estimate for the carrier concentration (p ment in a saturation temperature range, we can estin a thermopower measurement.

In some cases, conductivity is almost completely determined by the mobility term, i.e. it is easy to produce a free carrier, but hard to hop from one site to the next site. This would happen in cases, such as $Mn_{1-x}O$, which containes two oxidation states, Mn^{2+} (d^5) and Mn^{3+} (d^4).	If we only need to consider conductivity, we can also use a more simple model for thermopower (Heikes formula) $\alpha = \frac{k_B}{e} \left[\ln \frac{c}{1-c} + A \right],$ where <i>c</i> is the relative amount of the lower oxidation state ions (Mn ²⁺) and (1-c) is the relative amount of the higher oxidation state ions (Mn ³⁺). In this case the sign works out correctly, i.e. we get an <i>n</i> -type material when the higher oxidation state is predominant. This formula works well for La _{1-x} Sr _x CrO ₃ , where the calculated hole concentration (1-c) correlates well with the Sr content <i>x</i> .
In this case conductivity is given by $\sigma = c(1-c)\frac{e^2a^2\nu}{k_BT}e^{\frac{-E\mu}{k_BT}},$ where c and (1 - c) are the relative proportions of each oxidation state present in the crystal. The product, $c(1-c)$ is the probability that a jump to a neighboring site can take place, a is the lattice constant and ν is a hopping frequency. The exponential factor is the probability that a jump will take place. This works in e.g. La _{1-x} Sr _x CrO ₃ , where $\nu \approx 10^{13}$ Hz and $E_{\mu} = 0.11$ eV. 13	14
	An electron in the sample is subjected to an electric field and a magnetic field. The force on an electron is
Hall effect Another way of measuring the carrier mobility is to use the Hall effect.	$f = -eE - ev \times B.$ Current $J = -nev$ can be written in terms of electron momentum
Figure removed due to copyright restrictions	$\mathbf{J} = -ne\mathbf{v} = -\frac{ne}{m_e}\mathbf{p},$ where m_e is the electron mass, n is electron density and \mathbf{p} is the average electron momentum. In relaxation-time approximation the momentum change is $\frac{d\mathbf{p}(t)}{dt} = -\frac{\mathbf{p}(t)}{\tau} + \mathbf{f}(t).$ For our Hall effect calulation this means that $\frac{d\mathbf{v}}{dt} = -\frac{e}{m_e}\mathbf{E} - \frac{e}{m_e}\mathbf{v} \times \mathbf{B} - \frac{\mathbf{v}}{\tau}.$
Singleton01 p.134 Assume that the magnetic field is in the <i>z</i> -direction, $\mathbf{B} = (0, 0, B)$, and current flows in the <i>x</i> direction. Electric field is in the sample plane, $\mathbf{E} = (E_x, E_y, 0)$. Sample thickness is <i>s</i> , width is <i>w</i> .	In steady state, the derivative of v is zero and thus the two components of electron velocity are $\begin{aligned} v_x &= -\frac{e\tau}{m_e}E_x - \omega_c \tau v_y \\ v_y &= -\frac{e\tau}{m_e}E_y + \omega_c \tau v_y, \\ \end{aligned} $ where $\omega_c = eB/m_e$ is the classical cyclotron resonance frequency. 16

In steady state no current flows in the y-direction and we have $\frac{E_y}{E_x} = -\omega_c \tau.$	The Hall measurement thus tells us what the carrier type is $(n \text{ or } p)$ and what the carrier concentration is. If the Hall coefficient is combined with conductivity measurement, we get the Hall mobility	
Combining this, the current density $J_x = -env_x$ and the expression for v_x , we have	$\sigma R_{\rm H} = -\mu_e$	
$R_{H} \equiv \frac{E_y}{J_x B} = -\frac{1}{ne},$	for <i>n</i> -type and	
where $R_{\rm H}$ is the Hall coefficient. This applies to electrons (simple metals).	$\sigma R_{H} = +\mu_h$	
If we have several types of carriers, as in semiconductors or oxides, we would get	for <i>p</i> -type.	
$R_{\rm H} \equiv \frac{E_y}{J_x B} = \frac{1}{ e } \frac{p\mu_{hh}^2 - n\mu_c^2}{(n\mu_c + p\mu_{hh})^2}$	Hall mobility of reduced TiO ₂ . In	ı
For a purely n -type semiconductor,	oxides there are occasional difficulties	5
$R_{H} = -\frac{1}{ne}$	Figure removed due to copyright restrictions where the Hall coefficient changes sign	, 1
and for a purely p -type material	at the Neel temperature, although the	2
$R_{H} = +\frac{1}{pe}.$	may also be different from drift mobil- ity, seen in reduced LiNbO ₃ .	-
17	Cox92 p.168	18
		10
		10
In general, mobilities are very low in oxides, which means that scattering is very rapid and the average mean-free path is comparable to the lattice spacing.	Where does the activation energy come from?	25
In general, mobilities are very low in oxides, which means that scattering is very rapid and the average mean-free path is comparable to the lattice spacing. Measured mobilities can also suggest much larger effective masses than would be expected from band-structure calculations, because carriers can interact with	Where does the activation energy come from? An isolated carrier will polarize the lattice around itself. The total polarization, discussed last time, is divided into two parts	as
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Trapping thus happens when bands are narrow. This is quite typical of <i>d</i> -bands in transition metals. Polarons can only move from site to site by a hopping mechanism.	Even when bands are wide enough for carriers to delocalize (no small polarons), it is still possible for a carrier to polarize a larger region around itself. This is known as a <i>large polaron</i> . In this case a carrier is not trapped, but the apparent effective mass will be much larger. The mobility of small polarons increases with temperature due to faster hopping over a barrier. The mobility of large polarons decreases due to strong scattering from thermally-excited lattice vibrations.
This is where the mobility activation energy comes from in some oxides. If there is significant orbital overlap between neighboring sites, the barrier for hopping could be lowered (band is wider). It is possible to have a delocalized charge over two neighboring lattice sites.	22
	Kuänan Mink natation.
	Kroger-Vink notation:
Point defects in oxides It is easy to understand that oxygen loss in a d^0 insulator, e.g. TiO_{2-x} produces extra electrons and leads to <i>n</i> -type behavior. In order to understand such effects better, we use a <i>point defect model</i> .	Kroger-Vink notation: The way that we look at defects or impurities depends on what we want to understand. For spectroscopic purposes, for example, we might look at a Cr^{3+} impurity replacing Mg ²⁺ in MgO, Al ³⁺ in Al ₂ O ₃ , or Ti ⁴⁺ in TiO ₂ in exactly the same way.
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 Point defects in oxides It is easy to understand that oxygen loss in a d⁰ insulator, e.g. TiO_{2-x} produces extra electrons and leads to <i>n</i>-type behavior. In order to understand such effects better, we use a <i>point defect model</i>. The basic types of defects are vacancies: an atom is missing from a lattice site interstitial: an extra atom occupies an iregular position Frenkel: vacancy plus interstitial of the same ion 	Kroger-Vink notation: The way that we look at defects or impurities depends on what we want to understand. For spectroscopic purposes, for example, we might look at a Cr^{3+} impurity replacing Mg^{2+} in MgO , Al^{3+} in Al_2O_3 , or Ti^{4+} in TiO_2 in exactly the same way. For semiconductors theer are big differences between these cases. Cr^{3+} replaces Al^{3+} without producing any extra carriers. When it replaces Mg^{2+} , we must have a compensating negative charge in the crystal. When Ti^{4+} is replaced, we need an extra hole or defect. <i>Kröger-Vink notation</i> is used to mark the difference between the ionic charge and the charge that it replaces. Thus Cr^{3+} is neutral when replacing Al^{3+} , positive when replacing Mg 2+ and negative when replacing Ti^{4+} .

Notation:	A perfect lattice is marked with 0. The formation of a Schottky defect would be
Li [•] Interstitial Li ⁺ . The [•] marks a positive charge	$0 = \Box_{O}^{\bullet \bullet} + \Box_{M}''$
Li_{Mn}^{\dagger} Li ⁺ replacing Mn ²⁺ , giving a negative charge	and this shows the generation of intrinsic carriers
Cr_{AI}^{x} Cr_{AI}^{3+} replacing AI ³⁺ , no change of charge	$0 = e' + h^{\bullet}$
$ \begin{array}{ccc} & O^{2-} & \text{vacancy with no electron present, net } +2 & \text{charge} \\ O^{2-} & \text{vacancy with one trapped electron} \\ & \text{also marked as F}^+ & \text{center} \\ O^{2-} & \text{vacancy with two trapped electrons (F center)} \\ \end{array} $	Defects have the role of introducing carriers into a semiconducting oxide. Defects may also provide conductivity directly, by migrating in the lattice (ionic conductivity).
In this notation electrons would be marked e' and holes h^{\bullet} . These may be seen in chemical reactions involving defects, as in	Defect diffusivities are much lower than the mobility of electrons or holes. Re-
$Li = Li_i^\bullet + e',$	member that conductivity is $\sigma = \sum N_{constraints}$
which represents the insertion of lithium into an oxide, giving an interstitial ${\sf Li}^+$	$O = \sum_{i} N_{i} e \mu_{i},$
and a conduction band electron.	i.e. a sum of all conduction mechanisms. Each term in the sum is a product of concentration and mobility. Electrons have much larger mobility and, if present,
$Li_2O = 2Li'_{Mn} + O_{O}^{x} + \Box_{O}^{\infty}$	will dominate the transport properties.
by an oxygen vacancy. 25	True ionic conductivity occurs in wide-gap insulators, such as ZrO_2 . 26
Ionic conductivity in ZrO_2 can be increased by doping with CaO or Y_2O_3 . These are used as solid electrolytes in oxygen sensors	Defect spectroscopy
Ionic conductivity in ZrO ₂ can be increased by doping with CaO or Y ₂ O ₃ . These are used as solid electrolytes in oxygen sensors $O_2(p_1) ZrO_2 \cdot CaO O_2(p_2)$	Defect spectroscopy The precise nature of defects in oxides is hard to analyze, since direct defect structure observation is not usually possible. Spectroscopic techniques are often
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1		
Optical absorption		Absoption energy
Figure removed due to copyright restrictions	This mechanism is common in lightly reduced WO ₃ , SrTiO ₃ , KTaO ₃ . This plot shows bipolaron absorption in WO ₃ (a) and defect absorption after illumina- tion (b). The power law dependence is only ap- proximate, so determining scattering mechanisms from this data is not pos- sible. The exponent is usually in the	Figure removal due to copyright extractions Figure removal due to copyright extractions The polaron discussion showed that the optical absorption energy should be related to the mobility activation energy. In a simple parabolic case $E_{\rm opt} = 4E_{\mu}$.
	range 23.	Optical absorption of defect states is often difficult due to the intense absorp-
Cox92 p.188 In SrTiO ₃ , for example, several mech vacances \Box_{O}^{\bullet} . The absorption in W by carrier localization and strong loc WO ₃ is that the material does not s unpaired electrons). What appears t	nanisms contribute, including charged oxygen VO_3 is attributed to polarons, accompanied ocalized lattice distortions. What is odd in how an ESR signal in the dark condition (no co happen is that carrirs are trapped in pairs	tion. One way is to use very low defect concentrations. Other way is to use a different probing technique, such as electron energy loss spectroscopy (EELS), photoelectron spectroscopy (PES), or dielectric loss measurements.
at neighboring W ⁵⁺ sites. These are	e known as bipolarons. 29	30
Figure removed due to copyright restrictions	These figures show how the EELS spectrum chages with doping in Na _x WO ₃ in (a) and photoelectron spectra are shown in (b). Above $x = 0.3$ this material is metallic. We look what happens below $x = 0.3$. EELS shows a broad loss peak with a maximum at around 1 eV. This is similar to the optical absorption. The PES result shows a peak that does not change in shape below $x = 0.3$. Only the intensity drops as the number of defects drops. The peak width is controlled by vibrational excitations at the W ⁵⁺ localization site of a polaron.	Carrier activity In order to get a conducting crystal by doping, we need to ionize the carriers. If we think of a localized carrier at a defect site, the 'hydrogen' model would give an ionization energy of $E_c = R(m * /m_0)/\epsilon_r^2$, where R is the Rydberg constant, 13.6 eV, m^* is the effective mass of a carrier, and ϵ_r is the relative dielectric constant of the material. Several problems here: • What effective mass to use? • How does the band structure affect the effective mass?
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If the effective mass of a carrier is enhanced by polaron formation, the static	Semiconductor-to-metal transitions:		
(low-frequency, slow) dielectric constant should be used.			
At the same time, the hydrogen model only applies when the impurity orbital radius	Semiconductors become completely metallic when heavily doped.		
$a_0 = a_{\rm H} \epsilon_r / (m / m_0)$			
is much larger than the lattice spacing. $a_{\rm H} = 53$ pm, is the hydrogenic Bohr radius. If $(m^*/m) = 10$, this requires a dielectric constant of over 100. This is true in SrTiO ₃ and KTaO ₃ . In SrTiO ₃ , $(m^*/m_0) = 12m_0$ and $\epsilon_r \approx 220$ at room temperature. This would give abinding energy of about 3 meV and less than 0.1 meV for KTaO ₃ .	Figure removed due to copyright restrictions		
This shows why $SrTiO_3$ becomes metallic at a rather low doping level of about 0.5% because the impurity orbitals start to overlap.	Cox92 p.193		
In other oxides, where ϵ is much smaller, the radius is also smaller, comparable to the metal-metal distance. Localization of a carrier in a small volume like this will create strong polarization effects which must be included in any ionization energy calculation. A hydrogen model would not be accurate.	Here we look what happens to heavily doped $SrVO_3$, i.e. $La_{1-x}Sr_xVO_3$ as a function of doping level x . As we see, the activation energy drops as the doping level increases. At $x = 0.2$ the activation energy becomes zero and we get a metal.		
Ionic model would be more accurate here, but it only handles holes in metal orbitals. Photoelectron measurements clearly show that holes are often at the oxygen site, instead of the metal. So an ionic model would also fail. 33	34		
Transition to metallic state in doped oxides			
Figure removed due to copyright restrictions	This can be seen in $Nb_{18-n}W_{8+n}O_{69}$		
	Figure removed due to copyright restrictions		
Even when there is no transition, as in Li-doped NiO, the activation energy still drops with doping. This could be due to increased dielectric constant due to the extra polarizability of the impurity states. Higher ϵ weakens carrier binding to the impurities.	Cox92 p.194		
In various heavily-doped oxides, the Arrhenius activation energy plot does not work.	The carrier concentration goes up with n . At highest n there is no temperature dependence, i.e. we have a metallic state. The $T^{1/4}$ exponent is a sign of a		
$\sigma = A e^{-\left(\frac{T_0}{T}\right)^{1/4}},$	variable range hopping mechanism of conduction.		

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37